Unique Monge type solutions in the multi-marginal optimal transport via graph theory

Adolfo Vargas-Jiménez
Joint work with Brendan Pass
PhD candidate in Mathematics-University of Alberta
vargasji@ualberta.ca

Abstract: This talk is centered around a multi-marginal optimal transport with surplus function

$$b(x_1, \dots, x_m) = \sum_{\{i,j\} \in P} x_i \cdot x_j,$$

where $P \subseteq Q := \{\{i, j\} : i, j \in \{1, 2, ...m\}, i \neq j\}$. We reformulate this problem using graph theory; in particular, each surplus is associated to a graph with m vertices whose set of edges is indexed by P. We then establish uniqueness and Monge solution results for two fairly general classes of surplus functions. These classes capture the Gangbo and Święch surplus $\sum_{1 \leq i < j \leq m} x_i \cdot x_j$, and the surplus $\sum_{i=1}^{m-1} x_i \cdot x_{i+1} + x_m \cdot x_1$ studied in an earlier work by the present authors, whose origin lies in the time discretization of Arnold's variational interpretation of the incompressible Euler equation.