Title: Separatrices of real analytic vector fields in dimensions two and three

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Abstract: Let X be a germ of real analytic vector field at $(\mathbb{R}^n, 0)$, $n \geq 2$. A *separatriz* is a real formal curve invariant by X. In any dimension, there are examples of vector fields without separatrices. This is the case, for instance, of center-focus type vector fields at $(\mathbb{R}^2, 0)$. In this lecture, we will present, for vector fields with isolated singularity in dimensions $n = 2$ and 3, sufficient conditions for the existence of separatrices. In dimension two, in the family of *topological real generalized curves* — i.e. vector fields without topological saddle-nodes in their desingularizations — a vector field X has a separatrix provided that either its *algebraic multiplicity* $\nu_0(X)$ or its *Milnor number* $\mu_0(X)$ is even. This generalizes a result by J.-J. Risler (2001). In dimension three, we prove that a vector field that is tangent to the levels of a non-constant germ of real analytic function – i.e. X has a real analytic *first integral* — admits a separatrix. In both cases, the proof relies on techniques of desingularization and indices of vector fields. We also give examples showing that the results are optimal, in the sense that we cannot assure the existence of analytic separatrices. The part in dimension two is a joint work with my Ph.D. student E. Cabrera. The part in dimension three is a joint work with F. Sanz (Universidad de Valladolid - Spain).