THE BLOW-UP PROBLEM FOR A SEMILINEAR PARABOLIC EQUATION WITH A POTENTIAL

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Abstract. Let Ω be a bounded smooth domain in $\mathbb{R}^N$. We consider the problem $u_t = \Delta u + V(x)u^p$ in $\Omega \times [0,T)$, with Dirichlet boundary conditions $u = 0$ on $\partial\Omega \times [0,T)$ and initial datum $u(x,0) = Mu_0(x)$ where $M \geq 0$, u_0 is positive and compatible with the boundary condition. We give estimates for the blow up time of solutions for large values of M. As a consequence of these estimates we find that, for M large, the blow up set concentrates near the points where $u_0^{p-1}V$ attains its maximum.

1. INTRODUCTION

In this paper we study the blow-up phenomena for the following semilinear parabolic problem with a potential

$$(1.1)\qquad \begin{cases} u_t = \Delta u + V(x)\, u^p & \text{in } \Omega \times (0,T), \\ u(x,t) = 0 & \text{on } \partial\Omega \times (0,T), \\ u(x,0) = Mu_0(x) & \text{in } \Omega. \end{cases}$$

First, let us state our basic assumptions. They are: Ω is a bounded, convex, smooth domain in $\mathbb{R}^N$ and the exponent p is subcritical, that is, $1 < p < (N+2)/(N-2)$. The potential V is Lipschitz continuous and there exists a constant $c > 0$ such that $V(x) \geq c$ for all $x \in \Omega$. As for the initial condition we assume that $M \geq 0$ and that u_0 is a smooth positive function compatible with the boundary condition. Moreover, we impose that

$$(1.2)\qquad M\Delta u_0 + \frac{\min_{x\in\Omega} V(x)}{2} M^p u_0^p \geq 0.$$

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We note that (??) holds for M large if Δu_0 is nonnegative in a neighborhood of the set where u_0 vanishes.

It is known that, and we will prove it later for the sake of completeness, once u_0 is fixed the solution to (??) blows up in finite time for any M sufficiently large. By this we understand that there exists a time $T = T(M)$ such that u is defined in $\Omega \times [0, T)$ and

$$\lim_{t \to T} \|u(\cdot, t)\|_{L^\infty(\Omega)} = +\infty.$$

The study of the blow-up phenomena for parabolic equations and systems has attracted considerable attention in recent years, see for example, [?], [?], [?], [?], [?], [?], [?], [?], [?] and the corresponding references. A good review in the topic can be found in [?]. When a large or small diffusion is considered, see [?], [?].

Important issues in a blow-up problem are to obtain estimates for the *blow-up time*, $T(M)$, and determine the spatial structure of the set where the solution becomes unbounded, that is, the *blow-up set*. More precisely, the blow-up set of a solution u that blows up at time T is defined as

$$B(u) = \{x / \text{ there exist } x_n \to x, \, t_n \nearrow T, \text{ with } u(x_n, t_n) \to \infty\}.$$

The problem of estimating the blow-up time and the description and location of the blow-up set has proved to be a subtle problem and has been addressed by several authors. See for example [?], [?] and the corresponding bibliographies.

Our interest here is the description of the asymptotic behavior of the blow-up time, $T(M)$, and of the blow-up set, $B(u)$, as $M \to \infty$. It turns out that their asymptotics depend on a combination of the shape of both the initial condition, u_0, and the potential V. Roughly speaking one expects that if $u_0 \equiv 1$ then the blow-up set should concentrate near the points where V attains its maximum. On the other hand if $V \equiv 1$ the blow-up set should be near the points where u_0 attains its maximum. Here we show that the quantity that plays a major role is $(\max_x u_0^{p-1}(x) V(x))^{-1}$.

Theorem 1.1. *There exists $\bar{M} > 0$ such that if $M \geq \bar{M}$ the solution of (??) blows up in a finite time that we denote by $T(M)$. Moreover, let*

$$A = A(u_0, V) := \frac{1}{(\max_x u_0^{p-1}(x) V(x))},$$

then there exist two positive constants C_1, C_2, such that, for M large enough,

$$-\frac{C_1}{M^{\frac{p-1}{4}}} \leq T(M)M^{p-1} - \frac{A}{p-1} \leq \frac{C_2}{M^{\frac{p-1}{3}}}, \tag{1.3}$$

and the blow-up set verifies,

$$u_0^{p-1}(a)V(a) \geq \frac{1}{A} - \frac{C}{M^{\gamma}}, \qquad \text{for all } a \in B(u), \tag{1.4}$$

where $\gamma = min(\frac{p-1}{4}, \frac{1}{3})$.

Note that this result implies that

$$\lim_{M\to\infty} T(M)M^{p-1} = \frac{A}{p-1}.$$

Moreover, it provides precise lower and upper bounds on the difference $T(M)M^{p-1} - \frac{A}{p-1}$.

We also observe that (**??**) shows that the set of blow-up points concentrates for large M near the set where $u_0^{p-1}V$ attains its maximum.

If in addition the potential V and the initial datum u_0 are such that $u_0^{p-1}V$ has a unique non degenerate maximum at a point $\bar{a}$, then there exist constants $c > 0$ and $d > 0$ such that

$$u_0^{p-1}(\bar{a})V(\bar{a}) - u_0^{p-1}(x)V(x) \geq c|\bar{a} - x|^2 \quad \text{for all } x \in B(\bar{a}, d).$$

Therefore, according to our result, if M is large enough one has

$$|\bar{a} - a| \leq \frac{C}{M^{\frac{\gamma}{2}}} \quad \text{for any } a \in B(u),$$

with $\gamma = min(\frac{p-1}{4}, \frac{1}{3})$.

Throughout the paper we will denote by C a constant that does not depends on the relevant parameters involved but may change at each step.

2. Proof of Theorem ??.

We begin with a lemma that provides us with an upper estimate of the blow-up time. This upper estimate gives the upper bound for $T(M)M^{p-1}$ in (**??**) and will be crucial in the rest of the proof of Theorem **??**.

Lemma 2.1. *There exist a constant $C > 0$ and $M_0 > 0$ such that for every $M \geq M_0$, the solution of (**??**) blows up in a finite time that verifies*

$$T(M) \leq \frac{A}{M^{p-1}(p-1)} + \frac{C}{M^{\frac{p-1}{3}}M^{p-1}}. \tag{2.1}$$

Proof: Let $\bar a \in \Omega$ be such that

$$u_0^{p-1}(\bar a)V(\bar a) = \max_x u_0^{p-1}(x)V(x),$$

L the constant of Lipschitz continuity of V, and K an upper bound for the first derivatives of u_0 and L.

In order to get the upper estimate let M be fixed and $\varepsilon = \varepsilon(M) > 0$ to be defined latter, small enough so all functions involved are well defined. Pick

$$\delta = \frac{\varepsilon}{2K},$$

then

$$V(x) \geq V(\bar a) - \frac{\varepsilon}{2} \quad \text{and} \quad u_0(x) \geq u_0(\bar a) - \varepsilon \quad \text{for all } x \in B(\bar a, \delta).$$

Let w be the solution of

$$\begin{aligned} & w_t = \Delta w + \left(V(\bar a) - \frac{\varepsilon}{2}\right) w^p && \text{in } B(\bar a,\delta) \times (0, T_w), \\ & w = 0 && \text{on } \partial B(\bar a,\delta) \times (0, T_w), \\ & w(x,0) = M(u_0(\bar a) - \varepsilon), && \text{in } B(\bar a,\delta) \end{aligned}$$

and T_w its corresponding blow up time. A comparison argument shows that $u \geq w$ in $B(\bar a,\delta) \times (0,T)$ and hence

$$T \leq T_w.$$

Our task now is to estimate T_w for large values of M. To this end, let $\lambda_1(\delta)$ be the first eigenvalue of $-\Delta$ in $B(\bar a,\delta)$ and let φ_1 be the corresponding positive eigenfunction normalized so that

$$\int_{B(\bar a,\delta)} \varphi_1(x)\,dx = 1.$$

That is,

$$\begin{cases} -\Delta\varphi_1 = \lambda_1(\delta)\varphi_1, & \text{in } B(\bar a,\delta), \\ \varphi_1 = 0 & \text{on } \partial B(\bar a,\delta). \end{cases}$$

Now, set

$$\Phi(t) = \int_{B(\bar a,\delta)} w(x,t)\varphi_1(x)\,dx.$$

Then $\Phi(t)$ satisfies $\Phi(0) = M(u_0(\bar{a}) - \varepsilon)$ and

$$\begin{aligned}
\Phi'(t) &= \int_{B(\bar{a},\delta)} w_t(x,t)\varphi_1(x)\,dx \\
&= \int_{B(\bar{a},\delta)} \left(\Delta w(x,t)\varphi_1(x) + \left(V(x_1) - \frac{\varepsilon}{2}\right) w^p(x,t)\varphi_1(x)\right)\,dx \\
&\geq -\lambda_1(\delta)\int_{B(\bar{a},\delta)} w(x,t)\varphi_1(x)\,dx \\
&\qquad + \left(V(\bar{a}) - \frac{\varepsilon}{2}\right)\left(\int_{B(\bar{a},\delta)} w(x,t)\varphi_1(x)\,dx\right)^p \\
&= -\lambda_1(\delta)\Phi(t) + \left(V(\bar{a}) - \frac{\varepsilon}{2}\right)\Phi(t)^p.
\end{aligned}$$

Let us recall that there exists a constant D, depending on the dimension only, such that the eigenvalues of the laplacian scale according to the rule $\lambda_1(\delta) = D\delta^{-2}$.

Now, we choose ε such that

$$\lambda_1(\delta) = D\delta^{-2} = D\left(\frac{\varepsilon}{2K}\right)^{-2} = \frac{\varepsilon}{2}(M(u_0(\bar{a}) - \varepsilon))^{p-1}.$$

So, ε is of order

$$\varepsilon \sim \frac{C}{M^{\frac{p-1}{3}}}.$$

Choose M_0 such that for $M \geq M_0$ the resulting ε is small enough. Then for any $M \geq M_0$ we have that

$$\Phi'(t) \geq (V(\bar{a}) - \varepsilon)\Phi(t)^p, \tag{2.2}$$

for all $t \geq 0$ for which Φ is defined.

Since $\Phi(0) = M(u_0(\bar{a}) - \varepsilon)$ and T_w is less or equal than the blow up time of Φ integrating (**??**) it follows that

$$\begin{aligned}
T_w &\leq \frac{1}{M^{p-1}(p-1)(V(\bar{a}) - \varepsilon)(u_0(\bar{a}) - \varepsilon)^{p-1}} \\
&\leq \frac{1}{M^{p-1}(p-1)V(\bar{a})u_0(\bar{a})^{p-1}} + \frac{C}{M^{\frac{p-1}{3}}M^{p-1}},
\end{aligned}$$

for all $M \geq M_0$. □

Now we prove a lemma that provides us with an upper bound for the blow up rate. We observe that this is the only place where we use hypothesis (**??**).

Lemma 2.2. *Assume* (**??**)*. Then there exists a constant C independent of M such that*

$$u(x,t) \leq C(T-t)^{-\frac{1}{p-1}}.$$

Proof: Let $m = \min_{x\in\Omega} V$. Following ideas of [?], set

$$v = u_t - \frac{m}{2} u^p.$$

Then v verifies

$$\begin{aligned} & v_t - \Delta v - V(x) p u^{p-1} v = \frac{m}{2} p(p-1) u^{p-2} |\nabla u|^2 \geq 0 && \text{in } \Omega \times (0,T), \\ & v = 0 && \text{on } \partial\Omega \times (0,T), \\ & v(x,0) = M\Delta u_0 + \Big(V(x) - \frac{m}{2} \Big) M^p u_0^p \geq 0 && \text{in } \Omega. \end{aligned}$$

Therefore $v \geq 0$ and hence

$$u_t \geq \frac{m}{2} u^p.$$

Integrating this inequality from 0 to T we get

$$u(x,t) \leq \frac{2^{\frac{1}{p-1}}}{(m(p-1)(T-t))^{\frac{1}{p-1}}} \equiv C(T-t)^{-\frac{1}{p-1}},$$

as we wanted to prove. □

We are now in a position to prove Theorem **??**.

Proof of Theorem ??: The idea of the proof is to combine the estimate of the blow-up time proved in Lemma **??** with local energy estimates near a blow-up point a, like the ones considered in [?] and [?], to obtain an inequality that forces $u_0^{p-1}(a)V(a)$ to be close to $\max_x u_0^{p-1} V$.

Let us now proceed with the proof of the estimates on the blow-up set. We fix for the moment M large enough such that u blows up in finite time $T = T(M)$ and let $a = a(M)$ be a blow up point. As in [?], for this fixed a we define

$$w(y,s) = (T-t)^{\frac{1}{p-1}} u(a + y(T-t)^{\frac{1}{2}}, t)|_{t=T(1-e^{-s})}.$$

Then w satisfies

$$w_s = \Delta w - \frac{1}{2} y \cdot \nabla w - \frac{1}{p-1} w + V(a + yTe^{-\frac{s}{2}}) w^p, \tag{2.3}$$

in $\cup_{s\in(0,\infty)} \Omega(s) \times \{s\}$ where $\Omega(s) = \Omega_a(s) = \{y \,:\, a + yTe^{-\frac{s}{2}} \in \Omega\}$ with $w(y,0) = T^{\frac{1}{p-1}} u_0(a + yT^{\frac{1}{2}})$. The above equation can rewritten as

$$w_s = \frac{1}{\rho} \nabla(\rho \nabla w) - \frac{1}{p-1} w + V(a + yTe^{-\frac{s}{2}}) w^p$$

where $\rho(y) = \exp(\frac{-|y|^2}{4})$.

Consider the energy associated with the "frozen" potential

$$V \equiv V(a),$$

that is

$$E(w) = \int_{\Omega(s)} \left(\frac{1}{2}|\nabla w|^2 + \frac{1}{2(p-1)} w^2 - \frac{1}{p+1} V(a) w^{p+1} \right) \rho(y)\, dy.$$

Then, using the fact that Ω is convex, we get

$$\frac{dE}{ds} \le -\int_{\Omega(s)} (w_s)^2 \rho(y)\, dy + \int_{\Omega(s)} (V(a + yTe^{-\frac{s}{2}}) - V(a)) w^p w_s \rho(y)\, dy.$$

Since $V(x)$ is Lipschitz and w is bounded due to Lemma **??**, then there exists a constant C depending only on N, p and V, recall that the constant in Lemma **??** does not depend on M, such that

$$\frac{dE}{ds} \le -\int (w_s)^2 \rho(y)\, dy + Ce^{-\frac{s}{2}} T \left(\int (w_s)^2 \rho(y)\, dy \right)^{1/2}.$$

Maximizing the right hand side of the above expression with respect to $\int (w_s)^2 \rho(y)\, dy$ we obtain

$$\frac{dE}{ds} \le Ce^{-s} T^2$$

and integrating is s we get

$$E(w) \le E(w_0) + CT^2. \tag{2.4}$$

Since w is bounded and satisfies (**??**), following the arguments given in [?] and [?], one can prove that w converges as $s \to \infty$ to a non trivial bounded stationary solution of the limit equation

$$0 = \Delta z - \frac{1}{2} y \cdot \nabla z - \frac{1}{p-1} z + V(a) z^p \tag{2.5}$$

in the whole $\mathbb{R}^N$.

Again by the results of [?] and [?], since p is subcritical, $1 < p < (N+2)/(N-2)$, the only non trivial bounded positive solution of (**??**) with $V(a) = 1$ is the constant $(p-1)^{-\frac{1}{p-1}}$. A scaling argument gives that the only non trivial bounded positive solution of (**??**) is the constant $k = k(a)$ given by

$$k(a) = \frac{1}{(V(a)(p-1))^{\frac{1}{p-1}}}.$$

Therefore, we conclude that

$$\lim_{s \to \infty} w = k(a)$$

if a is a blow-up point. Also by the results of [?], [?] we have

$$E(w(\cdot, s)) \to E(k(a)) \qquad \text{as } s \to \infty, \tag{2.6}$$

where

$$\begin{aligned} E(k(a)) = & \int \left(\frac{1}{2(p-1)} (k(a))^2 - \frac{1}{p+1} V(a) (k(a))^{p+1} \right) \rho(y)\, dy \\ = & (k(a))^2 \left(\frac{1}{2(p-1)} - \frac{1}{(p+1)(p-1)} \right) \int \rho(y)\, dy. \end{aligned}$$

By (**??**) and (**??**) we obtain that, if a is a blow-up point, then

$$E(k(a)) \le E(w_0) + CT^2.$$

where $w_0(y) = w(y,0) = T^{\frac{1}{p-1}} M u_0(a + yT^{\frac{1}{2}})$.

As u_0 is smooth, $y\rho(y)$ integrable, and $T^{\frac{1}{p-1}} M$ is bounded by Lemma **??**, there are constants C independent of a such that for $M \ge M_0$

$$\begin{aligned} E(w(\cdot,0)) &= \int_{\Omega(0)} \left(\frac{1}{2} |\nabla w_0(y)|^2 + \frac{1}{2(p-1)} w_0^2(y) \right) \rho(y)\, dy \\ & \quad - \int_{\Omega(0)} \left(\frac{1}{p+1} V(a) w_0^{p+1}(y) \right) \rho(y)\, dy \\ &\le \int_{\Omega(0)} \left(\frac{1}{2} (T^{\frac{1}{p-1}} M)^2 T |\nabla u_0(a)|^2 \right) \rho(y)\, dy \\ & \quad + \int_{\Omega(0)} \left(\frac{1}{2(p-1)} (T^{\frac{1}{p-1}} M u_0(a))^2 \right) \rho(y)\, dy \\ & \quad - \int_{\Omega(0)} \left(\frac{1}{p+1} V(a) (T^{\frac{1}{p-1}} M u_0(a))^{p+1} \right) \rho(y)\, dy \\ & \quad + CT^{\frac{3}{2}} + CT^{\frac{1}{2}}. \end{aligned}$$

Therefore, since $|\nabla u_0|$ is bounded,

$$\begin{aligned} E(w(\cdot,0)) &\le \int_{\Omega(0)} \left(\frac{1}{2(p-1)} (T^{\frac{1}{p-1}} M u_0(a))^2 \right) \rho(y)\, dy \\ & \quad - \int_{\Omega(0)} \left(\frac{1}{p+1} V(a) (T^{\frac{1}{p-1}} M u_0(a))^{p+1} \right) \rho(y)\, dy \\ & \quad + CT^{\frac{3}{2}} + CT^{\frac{1}{2}}. \end{aligned}$$

Or, since $T \le 1$ for M large

$$E(w(\cdot,0)) \le E(T^{\frac{1}{p-1}} M u_0(a)) + CT^{\frac{1}{2}}.$$

Hence we arrive to the following bound for $E(k(a))$

$$E(k(a)) \le E(w(\cdot,0)) + CT^2 \le E(T^{\frac{1}{p-1}} M u_0(a)) + CT^{\frac{1}{2}}. \tag{2.7}$$

Observe that if b is a constant then the energy can be written as

$$E(b) = \Gamma F(b),$$

where Γ is the constant

$$\Gamma = \int \rho(y)\,dy$$

and F is the function

$$F(z) = \left(\frac{1}{2(p-1)} z^2 - \frac{1}{p+1} V(a) z^{p+1} \right).$$

As F attains a unique maximum at $k(a)$ and $F''(k(a)) = -1$ there are α and β such that if $|z - k(a)| \leq \alpha$ then

$$F''(z) \leq -\frac{1}{2},$$

and if $|F(z) - F(k(a))| \leq \beta$ then

$$|z - k(a)| \leq \alpha.$$

From (**??**) we obtain

$$F(k(a)) \leq F(T^{\frac{1}{p-1}} M u_0(a)) + CT^{\frac{1}{2}}.$$

If M_1 is such that $C(T(M_1))^{\frac{1}{2}} = \beta$ then for $M \geq max(M_0, M_1)$

$$\beta \geq CT^{\frac{1}{2}} \geq F(k(a)) - F(T^{\frac{1}{p-1}} M u_0(a)).$$

Hence by the properties of F,

$$|k(a) - T^{\frac{1}{p-1}} M u_0(a)| \leq \alpha.$$

Therefore

$$CT^{\frac{1}{2}} \geq F(k(a)) - F(T^{\frac{1}{p-1}} M u_0(a)) \geq \frac{1}{4}(T^{\frac{1}{p-1}} M u_0(a) - k(a))^2.$$

So, using Lemma **??**,

$$\begin{aligned} k(a) - CT^{\frac{1}{4}} &\leq T^{\frac{1}{p-1}} M u_0(a) \\ &\leq \frac{u_0(a)}{(p-1)^{\frac{1}{p-1}} V^{\frac{1}{p-1}}(\bar{a}) u_0(\bar{a})} + \frac{C u_0(a)}{M^{\frac{1}{3}}} \qquad (2.8) \\ &= k(a)\theta(a) + \frac{C u_0(a)}{M^{\frac{1}{3}}}, \end{aligned}$$

where

$$\theta(a) = \left(\frac{u_0(a) V(a)^{\frac{1}{p-1}}}{u_0(\bar{a}) V(\bar{a})^{\frac{1}{p-1}}} \right)$$

and $\bar{a}$ is such that

$$u_0^{p-1}(\bar{a}) V(\bar{a}) = \max_x u_0^{p-1}(x) V(x).$$

Recall that

$$T \leq \frac{C}{M^{p-1}}.$$

Therefore, we get

$$k(a)(1-\theta(a)) \leq \frac{Cu_0(a)}{M^{\frac{1}{3}}} + \frac{C}{M^{\frac{p-1}{4}}} \leq \frac{C}{M^{\gamma}},$$

with $\gamma = min(\frac{p-1}{4}, \frac{1}{3})$.

As V is bounded we have that $k(a)$ is bounded from below, hence

$$(1-\theta(a)) \leq \frac{C}{M^{\gamma}},$$

that is,

$$\theta(a) \geq 1 - \frac{C}{M^{\gamma}}$$

and we finally obtain

$$u_0(a)V(a)^{\frac{1}{p-1}} \geq u_0(\bar{a})V(\bar{a})^{\frac{1}{p-1}} - \frac{C}{M^{\gamma}}. \tag{2.9}$$

This proves (??).

To obtain the lower estimate for the blow-up time observe that from (??) and the fact that $V(a) \geq c > 0$ we get

$$\begin{aligned} u_0(a) &\geq \frac{u_0(\bar{a})V(\bar{a})^{\frac{1}{p-1}}}{V(a)^{\frac{1}{p-1}}} - \frac{C}{V(a)^{\frac{1}{p-1}}M^{\gamma}} \\ &\geq \frac{u_0(\bar{a})V(\bar{a})^{\frac{1}{p-1}}}{V(a)^{\frac{1}{p-1}}} - \frac{C}{M^{\gamma}} \\ &\geq C > 0. \end{aligned} \tag{2.10}$$

Inequality (??) gives us

$$\frac{1}{(V(a)(p-1))^{\frac{1}{p-1}}} - CT^{\frac{1}{4}} \leq T^{\frac{1}{p-1}} M u_0(a).$$

Hence

$$\frac{1}{u_0(a)(V(a)(p-1))^{\frac{1}{p-1}}} - \frac{CT^{\frac{1}{4}}}{u_0(a)} \leq T^{\frac{1}{p-1}} M.$$

By (??) and $u_0^{p-1}(\bar{a})V(\bar{a}) = \max_x u_0^{p-1}(x)V(x)$ we get

$$\frac{1}{u_0(\bar{a})(V(\bar{a})(p-1))^{\frac{1}{p-1}}} - CT^{\frac{1}{4}} \leq T^{\frac{1}{p-1}} M$$

and using

$$T \leq \frac{C}{M^{p-1}}$$

we obtain

$$\frac{1}{u_0(\bar{a})(V(\bar{a})(p-1))^{\frac{1}{p-1}}} - \frac{C}{M^{\frac{p-1}{4}}} \leq T^{\frac{1}{p-1}} M$$

as we wanted to prove. □

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