ert.

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→ X = v.a. $\hat{X} = Pic^0(X)$ $L(H,\chi)$, H = forma Herm. $\operatorname{Im} H(\Lambda\times\Lambda) \subseteq \mathbb{Z}$

$\quad\quad \| $

$\{L(0,\chi) : \chi : \Lambda \to S^1 \text{ homo}\}$

$\quad\quad \|$

$\operatorname{Hom}_{\bar{\mathbb{C}}}(V,\mathbb{C})/\hat{\Lambda}$ $\quad \hat{\Lambda} = \{h \in \operatorname{Hom}_{\bar{\mathbb{C}}}(V,\mathbb{C}) : \operatorname{Im} h(\Lambda) \subseteq \mathbb{Z}\}$

→ L = div. amplio, $L = (H,\chi)$, H no deg.

$\varphi_L : X \to \hat{X}$, $x \mapsto H(x,\cdot) + \hat{\Lambda}$ $\quad (x \mapsto t_x^* L \otimes L^{-1})$

isogenia (sobre ker finito).

Prop: D es amplio $\Leftrightarrow$ $\ker \varphi_D$ es dim 0.

$(\overset{\|}{K(D)})$

→ Si $L = (H,\chi)$ no degenerado ⇒ $\operatorname{Im} H = \begin{pmatrix} 0 & D \\ -D & 0 \end{pmatrix}$ $D = \begin{pmatrix} d_1 & & \\ & \ddots & \\ & & d_g \end{pmatrix}$

→ Subvariedad abeliana (X, L) $\quad \begin{array}{ccc} Y & \overset{i}{\hookrightarrow} & X \\ \varphi_Y \downarrow & \circlearrowleft & \downarrow \varphi_L \\ \hat{Y} & \overset{\hat{i}}{\longleftarrow} & \hat{X} \end{array}$ $\quad L|_Y = i^* L$

"Y" $\quad \psi$ isog inversa

$N_Y \in \operatorname{End} X$, $N_Y = i\,\psi_{\varphi_Y}\,\hat{i}\,\varphi_L$ $\quad$ Imagen$(N_Y) = Y = i(Y)$

$(\ker N_Y)_0$ = comp conexa de $\ker(N_Y)$ sub. abeliana

Z

$0 \to Z \to \underset{g}{X} \to \underset{\dim Y}{Y} \to 0$ $\quad \dim Z = \dim X - \dim Y$.

¿dim $Z \cap Y$? $\quad y \in Y \cap Z \Rightarrow \ldots \Rightarrow y \in K(i^* L) \Rightarrow$ es finito!

(finito)

→ $Y \times Z \xrightarrow{\psi} X$

$(y,z) \mapsto y+z$ $\quad \ker \psi = \{(y,-y) : y \in Y \cap Z\} \simeq Y \cap Z$

finito

⇒ X es isógeno a $Y \times Z$

⇒ $X \sim X_1^{n_1} \times \ldots \times X_r^{n_r}$ (Teo isog. Poincaré) dim $X = 2$, Si $E \subsetneq X$ es sub. variedad abel.

$X \sim E \times E'$.

Ejl. $X = \{ y^2 = (x^3-1)(x^3-\lambda^6), \; \lambda \in \mathbb{C}, \lambda^6 \neq 1 \}$

$g_X = 2$, $j(x,y) = (x,-y)$, $\sigma_1(x,y) = \left(\frac{\lambda^2}{x}, \frac{\lambda^3 y}{x^3}\right)$, $\frac{t_4-t_1}{t_4-t_2} \frac{t_3-t_1}{t_3-t_2}$

$\sigma_2(x,y) = \left(\frac{\lambda^2}{x}, \frac{-\lambda^3 y}{x^3}\right)$ involuciones todas.

$\frac{t_5-t_1}{t_5-t_2} \cdot \frac{t_6-t_1}{t_6-t_2}$

$X/_{\langle j \rangle} = \mathbb{P}^1 \quad X/_{\langle \sigma_1 \rangle} = E_1 \quad X/_{\langle \sigma_2 \rangle} = E_2$.

$X \xrightarrow{\pi_i} E_i \qquad J(X) \xrightarrow{(\pi_i)_*} E_i \qquad E_i \xrightarrow{\pi_i^*} J(X)$ y en este caso π_i^* es inyectivo.

Si $E_i = \mathbb{C}/_{\langle 1, 2\tau_i \rangle} \Rightarrow \Pi = \begin{pmatrix} 1 & 0 & \tau_1+\tau_2 & \tau_2-\tau_1 \\ 0 & 1 & \tau_2-\tau_1 & \tau_1+\tau_2 \end{pmatrix}$ en una cierta base. (Λ)

$\sigma_1(A_1) = -A_2$

$\sigma_1(B_1) = -B_2$

$\sigma_2(A_1) = A_2$

$\sigma_2(B_1) = B_2$

$\sigma_1 = \begin{pmatrix} 0 & -1 & & \\ -1 & 0 & & 0 \\ & & 0 & -1 \\ 0 & & -1 & 0 \end{pmatrix} \quad j = \begin{pmatrix} -1 & & & \\ & -1 & & 0 \\ & & -1 & \\ 0 & & & -1 \end{pmatrix} \Rightarrow \sigma_2 = \begin{pmatrix} 0 & 1 & & \\ 1 & 0 & & 0 \\ & & 0 & 1 \\ 0 & & 1 & 0 \end{pmatrix}$

$JX = \mathbb{C}^2/\Lambda$

$(\pi_1)_*((z,w)+\Lambda) = z - w + \langle 1, 2\tau_1 \rangle \qquad (\pi_2)_*((z,w)+\Lambda) = z+w+\langle 1, 2\tau_2 \rangle$

$(\pi_1)^*((z + \langle 1, 2\tau_1 \rangle) = (z,-z) + \Lambda$

$(\pi_2)^*(z + \langle 1, 2\tau_2 \rangle) = (z,z) + \Lambda$.

$(\pi_1)^*(E_1) \cap (\pi_2)^*(E_2) = \pi_1^* E_1[2] = \pi_2^* E_2[2]$

$E_1 \times E_2 \longrightarrow JX$ grado 4.

$(x,y) \mapsto x + y$

otro automorfismo de X $\quad r(x,y) = (\omega x, y) \quad \omega^3 = 1$, $\omega \neq 1$

$\therefore r(A_1) = A_2 \quad r(A_2) = -A_1 - A_2 \quad r(B_1) = -B_1 + B_2$, $r(B_2) = -B_1$

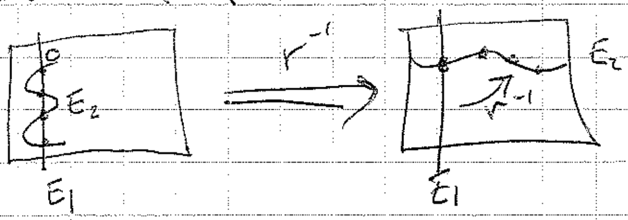
$\dot{r} = \begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$. Considerar $JX \xrightarrow{\pi_{1*}\times\pi_{2*}} E_1 \times E_2$

Interesante : $(\pi_1)_* \times (\pi_2 r)_* : JX \to E_1 \times E_2$
es isomorfismo!

Propi- $g(X) = 2$ $X \xrightarrow{j} X$ inv hiperel . $\mu \in \mathrm{Aut}X$
$\mu \neq j$, $|\mu| = 2$. Suponga $\exists \rho \in \mathrm{Aut}X$ orden impar
tq $\mu\rho = \rho^k \mu$ algún $k \neq 1$ y tq $JX \xrightarrow{\rho_*} JX$
no fija puntos de dos torsion .

$\Rightarrow JX \simeq X/_{j\mu} \times X/_{\rho^{-1}\mu\rho}$

$JX \xrightarrow{r} JX =$ (figura) $\xrightarrow{r^{-1}}$ (figura); $JX \xrightarrow{(\pi_1)_*} E_1$, $JX \to E_2$, $JX \xrightarrow{(\pi_2)_*} E_2$



r^{-1}(torsion 2 en E_1) = torsion 2 no en E_1
salvo 0