Robert.

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$$\mathbb{H}_g = \{ \tau \in M_{g\times g}(\mathbb{C}) \ : \ \tau^t = \tau \ , \ \operatorname{Im}\tau > 0 \}$$

$$Sp(2g,\mathbb{Z}) \curvearrowright \mathbb{H}_g$$

$$\| $$

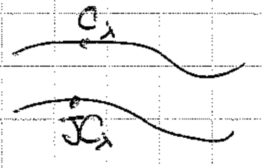
$$\{ M \in M_{2g\times 2g}(\mathbb{Z}) : M \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix} M^t = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix} \}$$

$$\underset{M}{\begin{pmatrix} A & B \\ C & D \end{pmatrix}} \cdot \tau = (A\cdot\tau + B)(C\tau + D)^{-1} \qquad AD - BC = I.$$

$$\therefore \quad \text{vapp}/\simeq \ \longleftrightarrow \ \mathbb{H}_g / Sp(2g,\mathbb{Z})$$

Si $\begin{pmatrix} A & B \\ C & D \end{pmatrix} \cdot \tau = \tau \implies \begin{pmatrix} A & B \\ C & D \end{pmatrix} \in \operatorname{Aut} X_\tau = \mathbb{C}^g / \mathbb{Z}^g + \tau\mathbb{Z}^g$.

Ej 1.



$C_\lambda \supseteq S_3 = \langle \sigma, r \rangle$ (ejemplo pasado).

$$S_{n+1} = \langle (1\,2\,3 \cdots n{+}1), (12) \rangle \xrightarrow{\ \rho\ } GL(n,\mathbb{Q})$$

$$(1 \cdots n{+}1) \mapsto \begin{pmatrix} -1 & -1 \cdots & -1 \\ & I & \begin{matrix} 0 \\ \vdots \\ 0 \end{matrix} \end{pmatrix}_{n\times n} = \sigma$$

$$(12) \mapsto \begin{pmatrix} \begin{matrix} 0 & 1 \\ 1 & 0 \end{matrix} & \\ & I \end{pmatrix} = \eta$$

repres. irreducible

para hacerlo simpléctico.

$$S_{n+1} \xrightarrow{\ \tilde{\rho}\ } Sp(2n,\mathbb{Z})$$

$$(1 \cdots n{+}1) \mapsto \begin{pmatrix} \sigma & 0 \\ 0 & \sigma^{-t} \end{pmatrix} \qquad \text{repr. simpléctica.}$$

$$(12) \mapsto \begin{pmatrix} \eta & 0 \\ 0 & \eta^{-t} \end{pmatrix}$$

Encontrar $\tau \in \mathbb{H}_n$ tq $\phi\cdot\tau = \tau \quad \forall \phi \in \tilde{\rho}(S_{n+1})$ ($\Leftarrow S_{n+1} \leq \operatorname{Aut}(X_\tau,$ con polarización

$$\therefore \quad \boxed{\begin{matrix} \sigma\tau\sigma^t = \tau \\ \eta\tau\eta = \tau \end{matrix}} \iff \tau = \lambda \underbrace{\begin{pmatrix} n & & -1 \\ & \ddots & \\ -1 & & n \end{pmatrix}}_{\tau_0} \qquad \lambda \in \mathbb{C}$$

Además, $\lambda z_0 \in \mathbb{H}_n \iff \lambda \in \mathbb{H}$. (2)

$\mathbb{Q}[S_{n+1}]$ $\quad 1 = e_1 + \dots + e_s \quad e_i \in Z(\mathbb{Q}[S_{n+1}]) \quad e_i^2 = e_i$

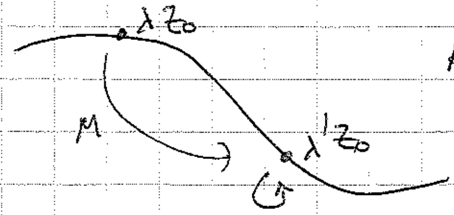
e_i irred.

sólo $e_2 \rightsquigarrow f_1$

$e_2(X_\tau) \sim B_2^n \Rightarrow B_2 = E$.

$\longrightarrow \mathcal{F} = \{ \lambda z_0 : \lambda \in \mathbb{H} \} \qquad \mathrm{Stab}\, \mathcal{F} = \{ M \in Sp(2g, \mathbb{Z}) : M \cdot \mathcal{F} = \mathcal{F} \}$

<u>Af</u>: El elemento general de $\mathcal{F}$ tiene $\mathrm{Aut} \simeq S_{n+1}$.



$\mathrm{Aut}\, X_{\lambda z_0} \simeq S_{n+1}$ $(n \neq 5\ ?)$

$\mathrm{Aut}\, S_6 \neq S_6 \qquad \mathrm{Aut}\, S_{n+1} \simeq S_{n+1}$, $n \neq 5$

$\Rightarrow \exists\, \alpha \in S_{n+1}$ tq $M^{-1} h M = \alpha h \alpha^{-1}\ \forall h \in S_n$

$\Rightarrow M\alpha$ conmuta con S_{n+1}

<u>Objetivo</u>: Encontrar $C_{Sp(2g,\mathbb{Z})} S_{n+1}$

<u>Lema</u>: $N = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \in Sp(2n, \mathbb{Z})$ está en $C_{Sp(2n,\mathbb{Z})} S_{n+1}$

$\iff A = mI,\ B = nB_0,\ C = rC_0,\ D = sI$

$B_0 = \begin{pmatrix} -g & & 1 \\ & \ddots & \\ 1 & & -g \end{pmatrix} = -z_0 \qquad C_0 = \begin{pmatrix} 2 & & 1 \\ & \ddots & \\ 1 & & 2 \end{pmatrix} = z_0^{-1}$ número.

$n \in \mathbb{Z}$, $\Gamma_0(n) = \{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2, \mathbb{Z}) ; \ c \equiv 0 \pmod n \}$

<u>Prop</u>: $C_{Sp(2n,\mathbb{Z})} S_{n+1} \simeq \Gamma_0(g+1)$.

$\begin{matrix} mI & -rz_0 \\ sC_0 & yI \end{matrix} \longmapsto \begin{pmatrix} m & r \\ -(g+1)s & y \end{pmatrix} \qquad \Gamma_0(g+1) \ltimes S_{n+1}$

$M\alpha \in \Gamma_0(g+1) \Rightarrow M \in \Gamma_0(g+1) S_{n+1} \Rightarrow \mathrm{Stab}\, \mathcal{F} = \Gamma_0(g+1) S_{n+1}$

Así Vapps con esta acción de S_{n+1} $/\simeq \iff \mathbb{H}/\Gamma_0(g+1)$ curva modular