Jacobianos

$C =$ curva suave proy. $/_{\mathbb{C}}$ (Sup. Riemann compacta).
$\Omega^1_C = \{$formas diferenciales holomorfas globales$\}$
$\dim_{\mathbb{C}} \Omega^1_C = g =$ género (C).

$$H_1(C,\mathbb{Z}) = \pi_1(C)/_{\text{comm.}} = \langle \alpha_1,\dots,\alpha_g,\beta_1,\dots,\beta_g\rangle_{\mathbb{Z}}.$$

$$\eta \in H_1(C,\mathbb{Z}),\quad \begin{aligned}\Omega^1_C &\longrightarrow \mathbb{C}\\ \omega &\longmapsto \int_\eta \omega\end{aligned} \qquad H_1(C,\mathbb{Z}) \hookrightarrow (\Omega^1_C)^*$$

Def 1: $J(C) := (\Omega^1_C)^* / H_1(C,\mathbb{Z})$ ~~toro~~ toro complejo.

$H_1(C,\mathbb{Z}) \times H_1(C,\mathbb{Z}) \longrightarrow \mathbb{Z}$ intersección, matriz $\begin{pmatrix} 0 & I \\ -I & 0\end{pmatrix}$.

$$\therefore\quad \begin{aligned} JC &\longrightarrow \mathbb{C}^g/\Lambda \\ \lambda + H_1(C,\mathbb{Z}) &\mapsto (\lambda(\omega_1),\dots,\lambda(\omega_g)) + \Lambda\end{aligned}$$

matriz período $\begin{pmatrix} \int_{\alpha_1}\omega_1 & \dots & \int_{\alpha_1}\omega_g \\ \vdots & & \vdots \\ \int_{\beta_g}\omega_1 & & \int_{\beta_g}\omega_g \end{pmatrix} = \Pi$.

Conclusión: JC es variedad abeliana, $\underline{\mathrm{Im} H}$: prod. de intersección.

Constr 2: $C \xrightarrow{\alpha_p} JC$, $p \in C$.
$$q \longmapsto \left(\int_p^q \omega_1,\dots,\int_p^q \omega_g\right) + \Lambda$$

$\Rightarrow$ α_p es 1-1.

$$\begin{aligned} \mathrm{Div}^0 C &\xrightarrow{\alpha} JC \\ \sum (p_i - q_i) &\mapsto \sum\left(\int_{q_i}^{p_i}\omega_1,\dots,\int_{q_i}^{p_i}\omega_g\right) + \Lambda \end{aligned}$$

Teo: α es sobre y $\ker\alpha = P\mathrm{Div}(C)$.

Cor: $JC \simeq \mathrm{Pic}^0(C)$.

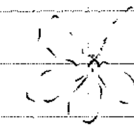
Ej: (1) $\mathbb{C}/\Lambda$, $\Omega^1 = \langle dz \rangle$

$$\alpha_0 : \mathbb{C}/\Lambda \to J(\mathbb{C}/\Lambda) \qquad \Rightarrow \quad \mathbb{C}/\Lambda \simeq J(\mathbb{C}/\Lambda).$$
$$z+\Lambda \mapsto \int_0^z dz + \underbrace{\tilde{\Lambda}}_{\Lambda} = z + \Lambda$$

← base explícita

$y^2 = x^6 - 1 : C \longrightarrow \hat{\mathbb{C}}$ $\qquad \pi_1(C) = \langle \beta_i - \delta_i : i \rangle$

$(x,y) \mapsto x$ $\qquad \Omega^1_C = \langle \frac{dy}{x^4}, \frac{dy}{x^5} \rangle$



otra base (Lange) $\Pi = \begin{pmatrix} 2i/\sqrt{3} & i/\sqrt{3} & 1 & 0 \\ i/\sqrt{3} & 2i/\sqrt{3} & 0 & 1 \end{pmatrix}$

→ Torelli $T : M_g \xrightarrow{\text{Periodo}} A_g$ es 1-1.

→ <u>Divisor Θ</u>: Θ divisor tal que $c_1(\Theta) = H$ en JC.

$$\mathcal{O}_{JC}(\Theta) = L(H, \alpha)$$

$\mathrm{Pic}^0 C \xrightarrow{\sim} \mathrm{Pic}^n C$, fijar $\deg E = n$.

$F \mapsto E + F$

$$\mathrm{Sym}^n C = C^n / S_n = \{ p_1 + \cdots + p_n : p_i \in C \}$$

$\rho_n : \mathrm{Sym}^n C \longrightarrow \mathrm{Pic}^n C$ es biracional con su imagen.

$\rho_n^{-1}(E) = |E|$ $\qquad n \le g$.

En particular, $\mathrm{Sym}^{g-1} C \longrightarrow \mathrm{Pic}^{g-1} C \simeq JC$

$W_{g-1} := \rho_{g-1}(\mathrm{Sym}^{g-1} C)$ es un divisor en JC, es irreducib

$W_{g-1} = \{ L \in \mathrm{Pic}^{g-1} C : |L| \neq \emptyset \} = \{ L \in \mathrm{Pic}^{g-1} C : h^0(L) \geq 1 \}$.

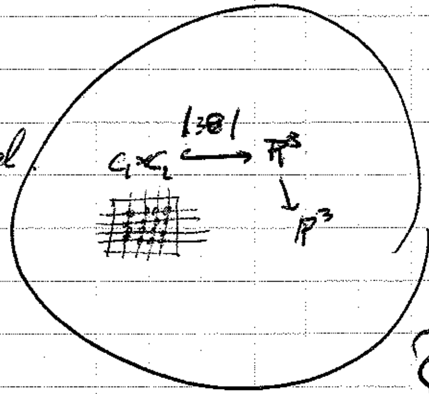
<u>Prop</u>: $\exists\, \eta \in \mathrm{Pic}^{g-1} C$ tq $\Theta + \eta = W_{g-1}$ → con la <u>polarización</u>

Si Θ es simétrico $\Rightarrow$ puedo tomar $2\eta \sim K_C$.

→ <u>Teo. singularidades de Riemann</u>:

$$\mathrm{mult}_L(W_{g-1}) = h^0(L).$$

<u>Prop</u>: $\dim \mathrm{Sing}\,\Theta = \begin{cases} g-4 & C \text{ no es hiperel.} \\ g-3 & \text{si lo es.} \end{cases}$



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