15/octubre/24

Superficies K3 $|_{\mathbb{C}}$ (Sebastián Torres)

Def.- X K3 si (i) $\omega_X \simeq \mathcal{O}_X$ (ii) $H^1(X, \mathcal{O}_X) = 0$. $\{\text{Sup K3 alg}\} \subsetneq \{\text{Sup K3}\}$

obs.- $\Omega^1_X \simeq T_X$.

Ej.- $X_4 \subset \mathbb{P}^3$ nonsingular, $X_4 = \{F=0\}$, $K_X = -4H + 4H|_X = 0$.

$$\cdots \to \underbrace{H^1(\mathcal{O}_{\mathbb{P}^3})}_{0} \to H^1(\mathcal{O}_X) \to \underbrace{H^2(\mathcal{O}(-4))}_{0} \Rightarrow H^1(\mathcal{O}_X) = 0.$$

Lo mismo con $X_{2,2,2}$, $X_{2,3}$

Ej.- $X \xrightarrow[\pi]{2:1} \mathbb{P}^2 \supset C$ grado 6 ramif. nonsingular $\qquad \pi_* \mathcal{O}_X = \mathcal{O} \oplus \mathcal{O}(-3)$, $H^1(\mathcal{O}_X) = 1$.

Ej.- A superficie abeliana, $A \simeq \mathbb{C}^2/\Lambda$ $\quad i : A \to A$ $\quad a \mapsto -a$.

(Kummer) $A \to A/\langle i \rangle \xleftarrow{\text{res. min.}} X = K3$?

$$\begin{tikzcd}
E_j \arrow[r, hook] \arrow[d] & \hat{A} \arrow[r, "\text{blow up}"] \arrow[d, "\pi", "2:1"'] & A \arrow[d, "2:1"] \\
E_j \arrow[r, hook] & X \arrow[r] & A/\langle i \rangle
\end{tikzcd}$$

$L^2 = \mathcal{O}(\sum E_j) \qquad \pi_* \mathcal{O}_{\hat{A}} = \mathcal{O}_X \oplus L^\vee$.

Sea $X = K3$.

Def.- $\mathrm{Pic}(X) \times \mathrm{Pic}(X) \to \mathbb{Z}$, $(L_1, L_2) \mapsto (L_1 \cdot L_2)$. **obs.-** $\mathrm{Pic}(X) = NS(X) = \mathrm{Num}(X)$

Def.- X var, $h^{p,q} := h^q(X, \Omega^p_X)$, $\Omega^p_X = \Lambda^p \Omega^1_X$, $\Lambda^0 \Omega^1 = \mathcal{O}_X$.

$h^{00} = h^0(\mathcal{O}_X) = 1 \quad \chi(\mathcal{O}_X) = 2 \quad c_2(X) = 24$, $h^{1,1} = 20$, $\rho(X) =$ rango $\mathrm{Pic}(X) \leq 20$

Referencia: "Lectures on K3 surfaces" (Huybrechts) Chapters 1.1-1.3