SESIÓN DE GEOMETRÍA: XXIX JMZS, 2016

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1. Torsion points on theta divisors

Robert Auffarth (U de Chile)

Let A be an abelian variety of dimension g, let $\Theta \subseteq A$ be an ample divisor on A and let $\Theta(n)$ be the number of n-torsion points that lie on Θ. An interesting problem is to find a bound for $\Theta(n)$. This number is related to the dimension of the space of quadrics that contain the Kummer variety of A, and is also involved in the number of components of the divisor

$$\{(A, \Theta) \in \mathcal{A}_g : \Theta \text{ has a singularity at a point of order } 2\} \subseteq \mathcal{A}_g,$$

where $\mathcal{A}_g$ is the moduli space of principally polarized abelian varieties. In this talk I will show where this number appears in problems related to abelian varieties, as well as recent progress that has been made on finding a bound for it.

2. Levi-flat hypersurfaces and their complement in complex surfaces

Carolina Canales González (U Paris-Sud)

We study analytic Levi-flat hypersurfaces in complex algebraic surfaces. These are real hypersurfaces that appear at the intersection of complex geometry and dynamics: on one side, they are the boundary of pseudoconvex domains, on the other side, they admit a foliation by holomorphic curves, called Cauchy-Riemann foliation (CR). We will see that, under some conditions on the dynamics of the CR foliation, the connected components of the complement of the hypersurface are modifications of Stein spaces. This allows us to extend the CR foliation to a singular algebraic foliation on the ambient complex surface.

3. Smooth rational projective varieties with a torus action of complexity 1 and Picard number 2

Anne Fahrner (U Tübingen)

We study smooth varieties with torus action and small Picard number. In the case of toric varieties, it is well known that the projectives spaces are the only smooth examples of Picard number one, and Kleinschmidt gave a description [18] of the smooth toric varieties having Picard number two. We go one step beyond toric varieties and study smooth projective rational varieties with a torus action of complexity one, i.e., the general torus orbit is of dimension one less than the variety. The case of Picard number one is basically settled by a result of Liendo and Süß [20]: the only non-toric examples are the smooth projective quadrics in dimensions three and four. In this talk we present a Cox ring-based approach [11] which yields an analogue of Kleinschmidt's description for complexity one and Picard number two. As a consequence we obtain in every dimension the smooth rational (almost) Fano varieties. It turns out that all the Fano examples are obtained via an iterated generalized cone construction from a series of smooth varieties of dimension at most seven.

4. Sums of squares in function fields of real surfaces and failure of a local-global principle

David Grimm (USACH)

The Pythagoras number $p(F)$ of a field F is defined as the smallest natural number n (if it exists), such that every sums of squares in the field is already a sum of n squares. For $F/\mathbb{R}$ an extension of finite type of transcendence degree d, the general upper bound $p(F) \leq 2^d$ was shown by Pfister in [23]. It is a widely open question when this so called Pfister-bound is optimal. In the case $d = 1$, i.e. when F is the function field of a real curve, the upper bound $p(F) \leq 2$ was already shown in [25] as a consequence of a local-global principle for isotropy of quadratic forms of rank 3 (and optimality of the Pfister-bound, i.e $p(F) = 2$, is immediate in this case).

Already in the case $d = 2$, i.e. when F is the function field of a real surface, the proof (obtained in [23] for arbitrary d) is not based on a local-global principle. It was recently shown in [5] that the local-global principle for quadratic forms of rank 4 fails when $\sqrt{-1} \in F$.

In the case where $\sqrt{-1} \notin F$, we will observe that $p(F) = 4$ implies the failure of the local-global principle for quadratic forms of rank 4. It is known for example that $p(\mathbb{R}(X,Y)) = 4$, as was shown for example in [8], but this is (to our knowledge) the only case of a function field of a real surface where optimality of the Pfister-bound is known. We recall some ideas of this proof, which involves methods from complex geometry that are beyond the scope of the presenter (but hopefully not beyond the scope of some interested members of the audience). An adaption of this method (if possible) might yield the optimality of the Pfister-bound for some non-rational function fields. We also present from [12] the general lower bound $p(F) \geq 3$ whenever $\sqrt{-1} \notin F$.

5. Computing Resolutions of Quotient Singularities

Simon Keicher (U Concepción)

This talk is a summary of [9]: Let $G \subseteq \mathrm{GL}(n)$ be a finite group without pseudo-reflections. We present an algorithm to compute and verify a candidate for the Cox ring of a resolution $X \to \mathbb{C}^n/G$, which does not require further information about the geometry. It contains a tropical and toric step and is based on [4]; it also uses methods of [4, 14]. We explain the algorithm by examples and show how to use our `Singular` implementation. As an application, we determine the Cox rings of resolutions $X \to \mathbb{C}^3/G$ for all $G \subseteq \mathrm{GL}(3)$ with the aforementioned property and of order $|G| \leq 12$. We close with a 4-dimensional example.

6. Derived symmetries of moduli spaces of sheaves on $K3$ surfaces

Sukhendu Mehrotra (PUC)

Let X be a $K3$ surface, and $X^{[g]}$ the Hilbert scheme of g points on it. It follows from results of Addington[1] and Markman-Mehrotra [21] that the derived category $D(X^{[g]})$ carries an exotic auto-equivalence constructed from the universal ideal sheaf. Addington has conjectured that any moduli space of sheaves on X should carry such a derived symmetry; in fact, it should arise from the same construction using the universal (twisted) sheaf. This was confirmed by him for a class of moduli spaces in recent work with Donovan and Meachan [2]. Here, we discuss another class of moduli spaces which was worked out jointly with Eyal Markman [21].

7. Exotic spaces and uniform rationality problem

Charlie Petitjean (U Talca)

An exotic space is an affine complex variety X of dimension $n > 2$ which is contractible (and therefore diffeomorphic to $\mathbb{R}^{2n}$) but which is not isomorphic to $\mathbb{C}^n$. One of the most famous examples is the Russell Cubic (see [19]). This variety is a smooth and rational hypersurface of $\mathbb{C}^4$ endowed with an algebraic torus action. An open question (see [7, 13]) is to know if every smooth and rational variety is uniformly rational, that is, does there exists an open neighborhood isomorphic to an open set in the affine space $\mathbb{C}^n$ for every $x \in X$. We will show how the algebraic torus action on X encoded by a geometrico-combinatorial presentation in the sense of Altmann and Hausen [3] can be used to prove the uniform rationality of the Russell cubic and other exotic spaces.

8. Generalized quaternion groups as groups of automorphisms of Riemann surfaces

Saúl Quispe (U de la Frontera)

Let Q_{2^n} be the generalized quaternion group of order 2^n, where $n \geq 3$. It is a well known fact that Q_{2^n} acts as a group of conformal automorphisms of some closed Riemann surface S [17].
In this talk we describe the actions of Q_{2^n} on S, with *triangular quotient* (i.e., the quotient orbifold S/Q_{2^n} has genus zero and it has exactly three cone points). We observe that such an action is unique, up to isomorphisms, and that it is purely non-free (i.e., every element of Q_{2^n} has fixed point). As a consequence, the *strong symmetric genus of* Q_{2^n} (i.e., the minimal genus of S admitting Q_{2^n} as a group of conformal automorphisms) equals the minimal genus of S where Q_{2^n} act *purely non-free*, this being 2^{n-2}. We also obtain that the symmetric genus of Q_{2^n} is $2^{n-2}+1$.

Also, we prove that there is a pseudo-real Riemann surface for which Q_{2^n} is the full group of conformal/anticonformal automorphisms.

9. Quasiplatonic curves with symmetry group $\mathbb{Z}_2^2 \rtimes \mathbb{Z}_m$ are definable over $\mathbb{Q}$

Sebastián Reyes Carocca (U de la Frontera)

Every compact Riemann surface S of genus $g \geq 2$ admitting a group G of automorphisms so that the quotient S/G has triangular signature can be defined over a finite degree extension of the field of $\mathbb{Q}$ (see, for example [26]). It is interesting to know, in terms of the algebraic structure of G, if S can in fact be defined over $\mathbb{Q}$. For example, this is the situation if G is either abelian or isomorphic to $A \rtimes \mathbb{Z}_2$, where A is an abelian group. On the other hand, if $G = \mathbb{Z}_p \rtimes \mathbb{Z}_q$ where $p, q > 3$ are prime integers, then S is not necessarily definable over the rational numbers [24]. In this talk, we shall study the situation when $G = \mathbb{Z}_2^2 \rtimes \mathbb{Z}_m$ with $m \geq 3$. We will see that S can be defined over $\mathbb{Q}$. Moreover, we describe explicit models for S, the corresponding groups of automorphisms and an isogenous decomposition of their Jacobian varieties as product of three Jacobians of hyperelliptic Riemann surfaces.

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