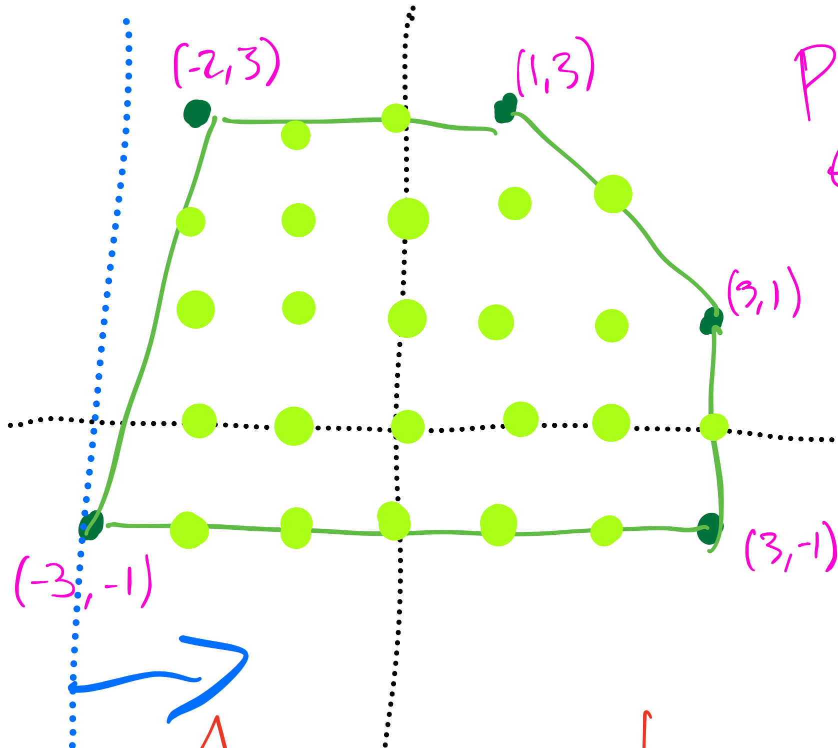
# Politopo

conjunto acotado
solución de sistema
de desigualdades
lineales

$$\{x \in \mathbb{R}^d \mid Ax \geq b\}$$




ptos extremos son enteros

$x \geq -3$

A L

$$\begin{bmatrix} -1 & -1 \\ -1 & 0 \\ 0 & 1 \\ 0 & -1 \\ 4 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \geq \begin{bmatrix} -4 \\ 3 \\ -1 \\ -3 \\ -11 \end{bmatrix}$$

$$|P \cap \mathbb{Z}^2| = 28$$

# Ejemplo

Encuesta

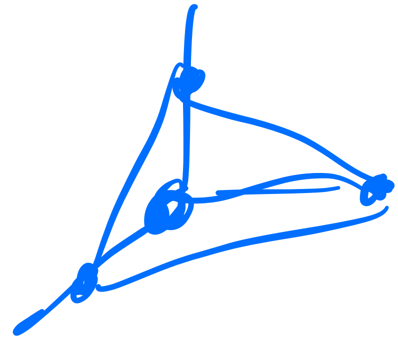
$n = 1000$

| | 200 | 200 | 100 | 200 | 200 | |
|---|---|---|---|---|---|---|
| 250 | | | | | | windows |
| 250 | | | | | | Ubuntu |
| 250 | | | | | | MacOS |
| 250 | | | | | | Debian |
| | Colo Colo | Cato | Audax | Calera | Everton | |

Sabiendo las marginales, cuantas tablas hay?

Nota: son integrales!

# Ejemplo



$$\operatorname{conv}\begin{bmatrix} 0 & a & 0 & 0 \\ 0 & 0 & b & 0 \\ 0 & 0 & 0 & c \end{bmatrix}$$

$$\|$$

$$\left\{ x \in \mathbb{R}^3 \;\middle|\; 0 \leq x \quad \frac{x_1}{a} + \frac{x_2}{b} + \frac{x_3}{c} \leq 1 \right\}$$

$$T(a,b,c)$$

¿Puntos enteros?

# Variedades tóricas todo sobre $\mathbb{C}$

<u>Def</u> $T \subset X$ denso y su acción extiende a $X$ $T \simeq (\mathbb{C}^*)^d$ * normal

$P$ reticular en $\mathbb{R}^d$ definimos $X$:

<u>Receta</u>

1. Inflar $(d-1)P$ $d \geq 2$

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2. para cada $t^{(-1,2)}$

$\vee$

$m \in (d-1)P$ $t_1^{-1} t_2^{2}$

$$\chi^m(t) = t^m$$

$$t \in (\mathbb{C}^*)^d$$

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3. $(\mathbb{C}^*)^d \longrightarrow \mathbb{P}^D$

$t \longrightarrow [\chi^{m_i}(t) : \dots]$

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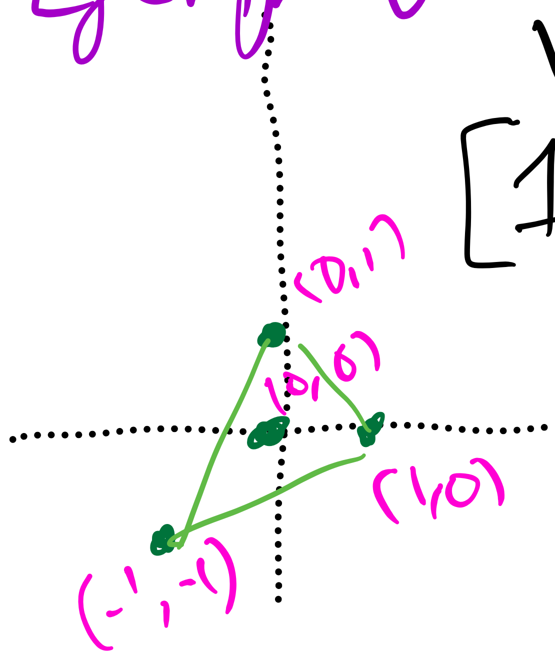
② tomar clausura

$$(\mathbb{C}^\times)^2$$
$$(t_1, t_2)$$
$$\Big\downarrow \text{ Clausura de}$$
$$[1 : t_1 : t_2 : t_1^{-1} t_2^{-1}]$$
$$\subseteq \mathbb{P}^3$$

Ejemplo



# Secciones globales

$X$ tórica

$D_P$ amplio

notar. * se vuelve muy amp mult. por $(d-1)$

$$H^0(X, \mathcal{O}(D_P)) = \{ f \in \mathbb{C}(X)^* \mid \mathrm{div}(f) + D_P \geq 0 \}$$

$$= \bigoplus_{m \in P \cap \mathbb{Z}^d} \mathbb{C} \cdot \chi^m$$

Conclusión

$$\boxed{h^0(X, D_P) = |P \cap \mathbb{Z}^d|}$$

# Ehrhart

$P$ reticular, $\dim = d$, $\subseteq \mathbb{R}^d$

$$\boxed{L_P(z) := |zP \cap \mathbb{Z}^d| \quad z \in \mathbb{N}}$$

# Teorema

1. $L_P(z)$ es polinomial*
2. grado dim(P)
3. $L_P(0) = 1$
4. L.T. es Vol(P)
5. $L_P(-z) = (-1)^d |zP^\circ \cap \mathbb{Z}^d|$

loadPackage AG

## Característica de Euler

$$\chi(D) = h^0(X,D) - h^1(X,D) + h^2(X,D) - h^3(X,D) + \dots$$

## Riemann-Roch asimptótico

$$\chi(zD) = \frac{D^d}{d!} z^d + O(z^{d-1})$$

$$\shortparallel$$

$$h^0(X,zD) - h^1(X,zD) + h^2(X,zD) - h^3(X,zD) + \dots$$

**Nota:** que sea polinomio de grado $\leq d$ no es dif

En general (para $D$ amplio)

$$h^i(X, zD) = 0, \quad i \geq 1, \quad z \gg 0 \qquad (*)$$

Demazure

$X$ tórica $\Rightarrow$ $(*)$ pasa $z \geq 0$

entonces $X(zD)$

$h^0(X, tD) = L_P(t)$

es polinomial

L.T. *

$$\boxed{\mathrm{Vol}(P) = \frac{D_P^d}{d!}}$$

$\chi(X, \mathcal{O}_X)$

''

$$L_p(\theta) = h^{\circ}(x, \theta_x)$$

$$= 1 .$$

* por ser completa

$\hat{x}$

$\chi(Y, -2D)$

$K$ canonical*

$Y$ $X$ C-M

Serre duality

$$h^i(X, -ZD) = h^{d-i}(X, ZD+K)$$

$$\chi(X, -ZD)$$

$$\|$$

$$(-1)^d\left[h^0(X, ZD+K) - \cancel{\cdots}\right]$$

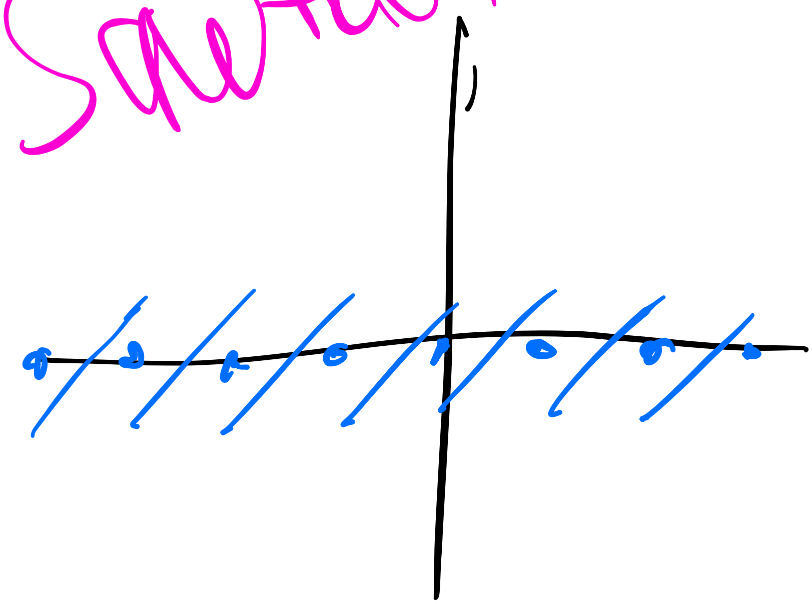
Kodaira

$$\overset{*}{=} (-1)^d \left| zP^{\circ} \cap \mathbb{Z}^d \right|$$

Y Dedekind?

$$((x)) = \begin{cases} \{x\} - \frac{1}{2} & x \notin \mathbb{Z} \\ 0 & x \in \mathbb{Z} \end{cases}$$

Sawtooth



# Suma de Dedekind

$$s(a,b) = \sum_{k=0}^{b-1} \left(\left(\frac{ka}{b}\right)\right)\left(\left(\frac{k}{b}\right)\right)$$

rel primos 2a2.

$$P = T(a, b, c)$$

$$L_P(z) = \frac{abc}{6} z^3 + \frac{(ab+ac+bc+1)}{4} z^2$$

$\boxed{\text{Pommersheim 93'}}$

$$+\left[\frac{3}{4}+\frac{a+b+c}{4}+\frac{1}{12}\left(\frac{bc}{a}+\frac{ca}{b}+\frac{ab}{c}+\frac{1}{abc}\right)\right.$$

$$-S(bc,a)-S(ca,b)$$

$$\left.-S(ab,c)\right]z$$

$$+1$$