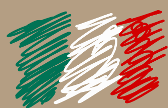
Scuola Italiana di geometria algebrica



«... mentre le curve algebriche (già composte in una teoria armonica) sono create da Dio, le superficie invece sono opera del Demonio.»

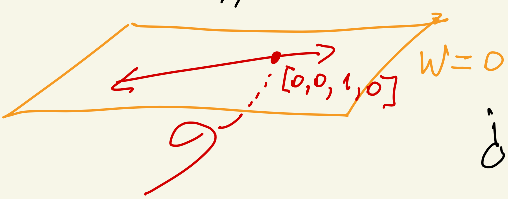
(Le superficie algebriche, (p. 464) F. Enriques)

iga
3/Junio/21
multi

¿Quedó claro el ejemplo hom. v/s ecuac. hom.?

$$I = (y - x^2, z - x^3), \quad V(I) \subseteq \mathbb{A}^3_{x,y,z}$$

$$\text{rojo} = \underset{\mathbin{/\!/}}{V(wy - x^2, zw^2 - x^3)} \overset{?}{\neq} \underset{\underset{V(I^*)}{\parallel}}{\overset{\cap}{V(I)^*}} \subset \overset{\cap}{\mathbb{P}^3_{x,y,z,w}}$$



¿Porqué? $y^3 - z^2 \in I$

$$\left(y^3 - z^2 \cancel{w}^{\,0} = 0 \Rightarrow y = 0. \right)$$

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§4.4 Espacio multiproyectivo.

$$\left[\mathbb{A}^n, \mathbb{P}^n, \text{productos } \underbrace{\mathbb{P}^{n_1} \times \dots \times \mathbb{P}^{n_r}}_{\text{multi-proy.}} \times \mathbb{A}^n \right]$$

Preg: ¿$V \subset \overset{\mathbb{A}^n}{\mathbb{P}^n}$, $W \subset \overset{\mathbb{A}^m}{\mathbb{P}^m}$ será que $V \times W$ es variedad $\underset{\text{afín}}{\text{proyectiva}}$?

Resp: $V \subset \mathbb{A}^n$, $W \subset \mathbb{A}^m$

$$V \times W \subset \mathbb{A}^n \times \mathbb{A}^m = \mathbb{A}^{n+m}$$

Ejs: $\overset{= V}{\{x = 0\}} \subseteq \mathbb{A}^1$ $\{y = 0\}^{= W} \subseteq \mathbb{A}^1 \Rightarrow \underset{\{(0,0)\}}{\underset{\parallel}{V \times W}} \subseteq \mathbb{A}^2$.

Volviendo a la pregunta...

$$V \times W \subseteq \underbrace{\mathbb{P}^n \times \mathbb{P}^m}_{\text{tendrá interpretación algebraica?}} \neq \mathbb{P}^{n+m}$$

Por otro lado, $V \times W \subseteq \mathbb{P}^n \times \mathbb{P}^m \subseteq \mathbb{P}^S$

✓ Segre

Resp.- $V \times W$ será var. proyectiva.

$$\mathbb{P}^N \times \mathbb{A}^M \subset \mathbb{P}^N \times \mathbb{P}^M \subset \mathbb{P}^\alpha$$

- Para $\mathbb{P}^n \times \mathbb{P}^m$: $k[X,Y] = k[x_0, x_1, \ldots, x_n, y_0, \ldots, y_m]$ y un $F \in k[X,Y]$ es <u>bi-forma</u> de grado (p,q) si es forma de grado p en X y q en Y.

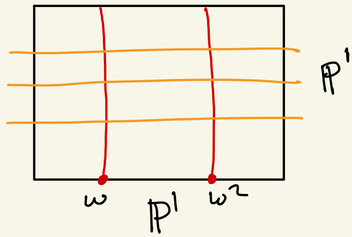
- Si $S \subset k[X,Y]$ conjunto de bi-formas
  $\Rightarrow V(S) := \{(x,y) \in \mathbb{P}^n \times \mathbb{P}^m : F(x,y) = 0, \forall F \in S\}$
  (Conj. algebraicos)

- Se define $I(V) = \{F \in k[X,Y] : F(x,y) = 0 \ \forall (x,y) \in V\}$
  $V \subseteq \mathbb{P}^n \times \mathbb{P}^m$ $\rightsquigarrow$ ideal bihomogeneo

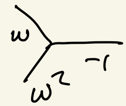
  y se definen variedades en $\mathbb{P}^n \times \mathbb{P}^m$ como los conj. alg. irreducibles.

• $K(V) = \left\{ z \in K_h(V) : z = \frac{\text{bi-forma}}{\text{bi-forma}} \text{ del mismo bi-grado} \right\}$

Ej.- $\mathbb{P}^1_{x,y} \times \mathbb{P}^1_{u,v} =$



$k = \mathbb{C}$



$k[x,y,u,v]$

Ψ

F bi-grado

$|| = C = \{ \underbrace{x^2+y^2+xy=0}_{\text{en } \mathbb{P}^1_{x,y}} \} \subset \mathbb{P}^1 \times \mathbb{P}^1$

$x^2+y^2+xy=0$

$t^2+t+1=0$

$\equiv \; D = \{ u^3 + uv^2 + u^2v + 7v^3 = 0 \}$

$E = \{ ux^2 + vy^2 = 0 \}$

$\{[0,1]\times[1,0]\} = E \cap \{ x = 0 \}$ $\qquad v \cdot y^2 = 0 \Rightarrow v = 0$.

$x=0$

y u es libre

$\left\{ \begin{array}{l} [1,1]\times[1,-1] \\ [1,-1]\times[1,-1] \end{array} \right\}$

$\shortparallel$

$E \cap \{ u = v \}$

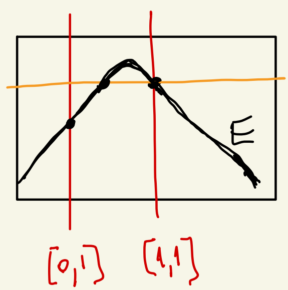
$ux^2 + vy^2 = 0$

$v(-x^2+y^2) = 0$

$x = y \quad x = -y$

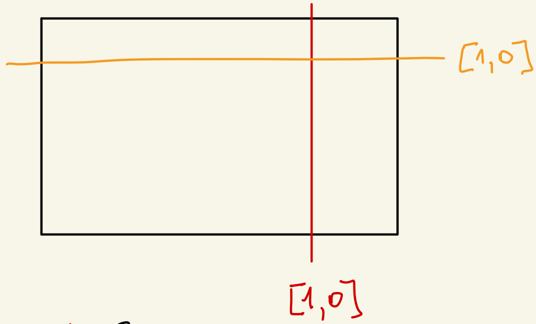
$E \cap \{ x = y \} = \{ [1,1]\times[1,-1] \}$

$[1,1]$



$u x^2 + v x^2 = 0$

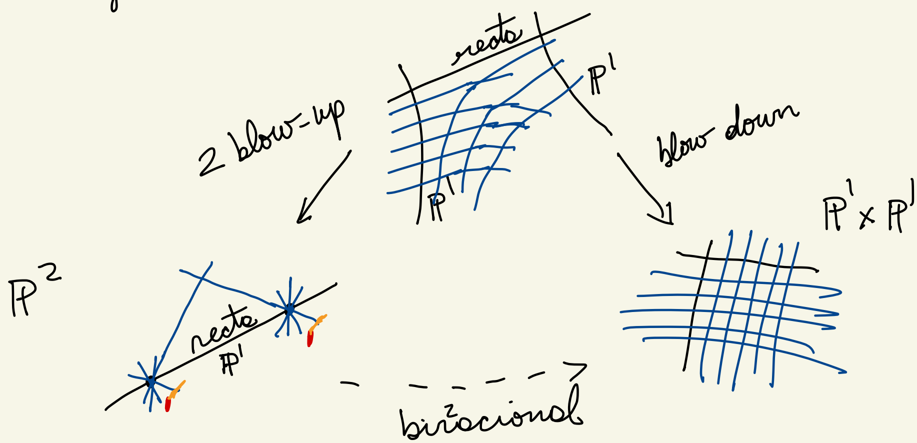
$x^2 (u+v) = 0 \qquad u = -v$

$\mathbb{P}^1 \times \mathbb{P}^1 =$ 

$\mathbb{P}^1 \times \mathbb{P}^1 \setminus \{$  $\} = \mathbb{A}^1 \times \mathbb{A}^1 = \mathbb{A}^2$

$\mathbb{A}^2 \subset \mathbb{P}^2$ $\qquad \mathbb{P}^2 \setminus \mathbb{A}^2 = L_\infty$ (1 recta)

¿Cómo voy de $\mathbb{P}^1 \times \mathbb{P}^1$ a $\mathbb{P}^2$?



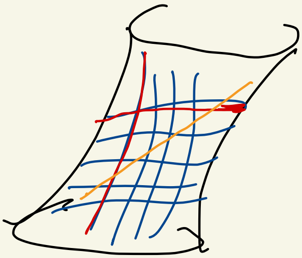
$\mathbb{A}^2$ se compactifica de ∞ maneras...

$\mathbb{P}^1 \times \mathbb{P}^1 \subset \mathbb{P}^3$

$$[x,y] \times [u,v] \longmapsto [\underset{z_0}{\underset{\shortparallel}{xu}}, \underset{z_1}{\underset{\shortparallel}{xv}}, \underset{z_2}{\underset{\shortparallel}{yu}}, \underset{z_3}{\underset{\shortparallel}{yv}}] \qquad \text{(Segre)}$$

¿Quién es $\mathbb{P}^1 \times \mathbb{P}^1$ en $\mathbb{P}^3$?

$$\mathbb{P}^1 \times \mathbb{P}^1 = \{ z_0 z_3 = z_1 z_2 \} \quad (\text{cuádrica})$$



$$\cap$$

$$\subset \mathbb{P}^3 \supset H' = \{\text{Plano general}\} \Rightarrow \text{no será reducible}$$

$$H = \{ z_0 + z_1 + z_2 + z_3 = 0 \}$$

$$\therefore \quad \{ xu + xv + yu + yv = 0 \}$$

$$\| $$

$$\{ x(u+v) + y(u+v) = 0 \}$$

$$\| $$

$$\{ (x+y)(u+v) = 0 \} =$$

¿Cómo represento $E = \{ ux^2 + vy^2 = 0 \} \subset \mathbb{P}^1 \times \mathbb{P}^1$ en $\mathbb{P}^3$?

$$z_0 = ux \quad z_1 = vx \quad z_2 = yu \quad z_3 = yv$$

$$E = \left\{ \begin{array}{l} u^2x^2 + uvy^2 = 0 \\ vux^2 + v^2y^2 = 0 \end{array} \right\}$$

$$= \left\{ \begin{array}{l} z_0^2 + z_2 z_3 = 0 \\ z_1 z_0 + z_3^2 = 0 \end{array} \right\} \subset \{ z_0 z_3 = z_1 z_2 \}$$

FIN