SGA UC

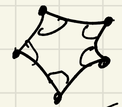
niec 4

26/10/20

Inoc. 4 :

La situación era

W-superficie (Moderno)

$$X_0 = X = $$  $$\subset \mathcal{X} \supset X_t = \text{sup. suave proy.}$$

$\frac{1}{n_i^2}(1, n_i a_i - 1)$

$\parallel$ sup proy. con solo sing. Wahl

$\mathcal{X} \to \mathbb{D}$, 0, t

$K_{\mathcal{X}}$ $\mathbb{Q}$-Cartier $\Leftrightarrow$ $\mathbb{Q}$-Gorenst.

$K^2_{X_t} = \text{const.}$

$\chi(\mathcal{O}_{X_t})$, $\chi_{top}(X_t)$, $q(X_t)$, $p_g(X_t)$

(1) $\mathrm{Pic}(X) \subset \mathrm{Pic}(X_t)$, cuando $\boxed{p_g = q = 0}$.

[Manetti, Lemma 2, Crelle]

(2) Asumir que $H_1(X_t) = 0$ y, entonces se espera que:

$$0 \to \mathrm{Pic}(X_t) \to Cl(X) = H_2(X) \to \oplus\, \mathbb{Z}/n_i \to 0$$

Si más aun $\pi_1(X)=\{1\}$, entonces

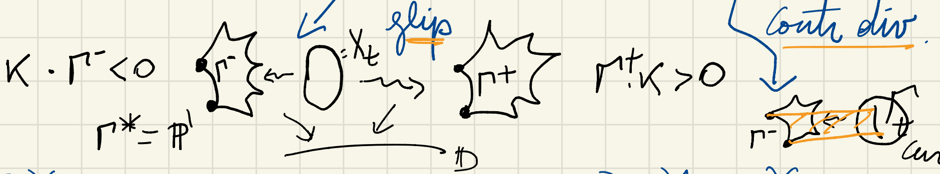
$$\mathrm{Pic}(X) \subset \mathrm{Pic}(X_t) \subset Cl(X).$$

$\prod n_i^2$ $\qquad$ $\prod n_i$

MMP explícito: $X_0 \subset \mathcal{X}_0$ $\downarrow$ $0 \in \mathbb{D}$ $\qquad$ K_{X_0} no neg $\longrightarrow$ (1) $X_0 \subset \mathcal{X}$ es degen suave $\to \mathbb{D}$ (X_0 suave) y $X_0 = \mathbb{P}_C(\mathcal{E})$.

$\Rightarrow$ Ej: $\mathbb{F}_1 \rightsquigarrow \mathbb{F}_3$

(2) $\mathbb{P}^2 \rightsquigarrow X_0 = \mathbb{P}(a^2, b^2, c^2)$ o suav. parcial de $\uparrow$



$$X_0 \subset \mathcal{X}_0 \dashrightarrow X_1 \subset \mathcal{X}_0 \dashrightarrow \cdots \dashrightarrow X_m \subset \mathcal{X}_m$$

$$\searrow \quad \downarrow \quad \swarrow$$

$$\mathbb{D}$$

K_{X_m} es neg, y

X_m es único.

Lado positivo ($\Gamma^+ \subset X^+ \subset \mathfrak{X}^+$):

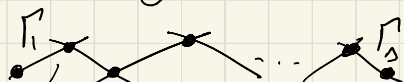
Sing T : $\frac{1}{dn^2}(1, dna-1)$ $\quad$ mcd$(n,a)=1$

Def.- Fijar una sing. $\frac{1}{m}(1,q) \in Y$ (germ de sup.).

Una P-resolución es una resolución parcial $X \xrightarrow{f} Y$ (f propia birracional) con X con sólo sing. de tipo T o ADE y K_X es f-amplio.

$d=1$ $\Updownarrow$ Wahl.

$$X \longrightarrow Y$$
$$\cup$$



Γ_1 $\quad$ Γ_s $\longrightarrow \frac{1}{m}(1,q)$

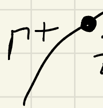
sing T o ADE

Cadena de $\mathbb{P}^1$

$\Gamma_i \cdot K_X > 0$.

$3 - \frac{1}{4} = \frac{11}{4}$

Ej.-



Γ^+ $\quad$ $\frac{1}{4}(1,1)$ $= X$

P-resolución de

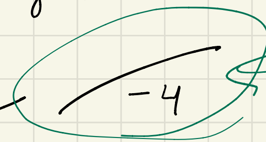
$Y = \frac{1}{11}(1,4)$

$K_X \cdot \Gamma^+ = 1 - \frac{1}{2} = \frac{1}{2} > 0$.

Kollár – Shepherd-Barron 1988 : $\#\{$P-resoluciones de $\frac{1}{m}(1,q)\} =$ finito.

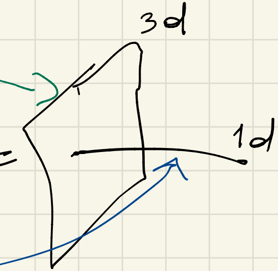
Componentes de $\mathrm{Def}\left(\frac{1}{m}(1,q)\right) \xleftrightarrow{1-1}$ P-resoluciones

Ej.- $\frac{1}{4}(1,1) \longleftarrow$



$\frac{1}{4}(1,1) \xleftarrow{\cong} \bullet\,\frac{1}{4}(1,1)$

$\Longleftrightarrow \quad \mathrm{Def}\left(\frac{1}{4}(1,1)\right) =$



$\frac{4}{3} = 2 - \cfrac{1}{2 - \cfrac{1}{2}}$

$[2,2,2]$

$[2,1,2] \quad [1,2,1]$

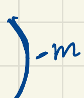
P-resoluciones $\xleftrightarrow{1-1}$ $K\left(\frac{m}{m-q}\right)$

$K\left(\frac{m}{m-q}\right) = \{[k_1,\dots,k_s]=0 : \ 1 \leq k_i \leq a_i\}$

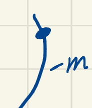
$\frac{m}{m-q} = [a_1,\dots,a_s]$

Lado positivo: P-resoluciones extremales, P-resol con un $\mathbb{P}^1$ y solo sing Wahl. $\Longleftarrow$ $[a_1,\dots,a_i-1,\dots,a_j-1,\dots,a_s]=0$.

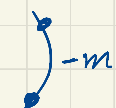
Γ^+ :



$, m \geq 3$



$, m \geq 2$



$, m \geq 1$

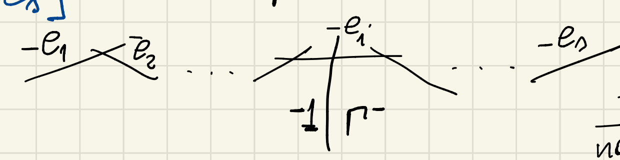
$$\Gamma^{+} \cdot K_X = \frac{\delta}{n_1 n_2} > 0 \quad , \quad \Gamma^{+2} = -\frac{\Delta}{n_1^2 \cdot n_2^2} < 0$$

n_i = índices (suave $\iff n_i = 1$) $\quad (\Delta = m)$.

<u>Lado negativo</u> $(\Gamma^- \subset X^- \subset \mathcal{X}^-)$:

mk1A: $\quad X^- \supset \Gamma^-$, $\frac{1}{n^2}(1, na-1) \longrightarrow \frac{1}{m}(1,q) \in Y \quad , \quad K_X \cdot \Gamma^- = -\frac{\delta}{n} < 0$

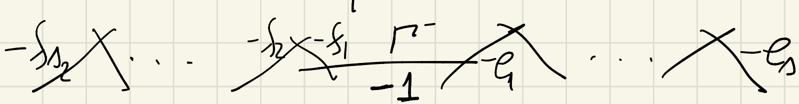
$[e_1, \dots, \bar{e}_i, \dots, e_s]$ $\uparrow$ $\qquad \Gamma^{-2} = -\frac{m}{n^2} < 0$



$$\frac{n^2}{na-1} = [e_1, \dots, e_s]$$

mk2A: $\quad X^- \supset \Gamma^-$, $\binom{n_2}{a_2}$, $\binom{n_1}{a_1} \longrightarrow \frac{1}{m}(1,q) \in Y \quad , \quad K_X \cdot \Gamma^- = -\frac{\delta}{n_1 n_2} < 0$

$[f_{s_2}, \dots, f_1] - [e_1, \dots, e_s]$ $\uparrow$ $\qquad \Gamma^{-2} = -\frac{m}{n_1 n_2} < 0$



$$\frac{n_2^2}{n_2 a_2 - 1} = [f_1, \dots, f_{s_2}] , \dots$$

Si K_X no es neg $\Rightarrow$ aparece una mk1A o mk2A, y puede ser flip o contr. div.

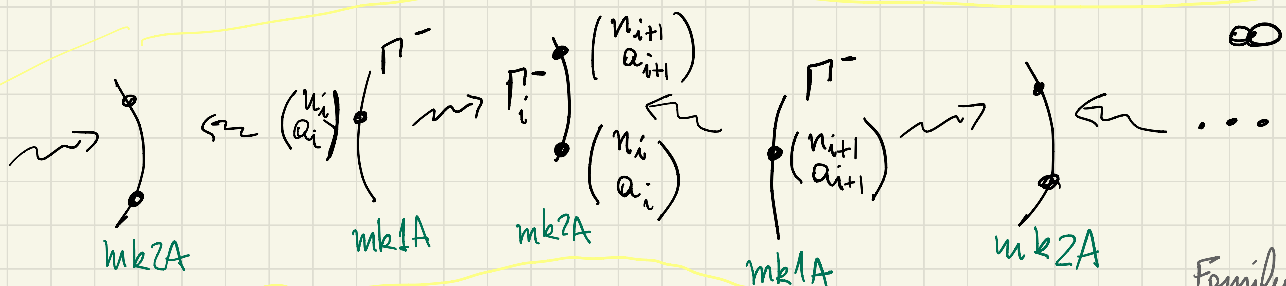
mk1A  $\longrightarrow \frac{1}{m}(1,q)$

$\xrightarrow{\text{flip}}$ $\exists$ P-resol. extremal de $\frac{1}{m}(1,q)$

$\xrightarrow{\text{contr. div}}$ $\frac{1}{m}(1,q)$ tiene que ser Wahl.

Teo: A lo más $\exists$ 2 P-resol. extremales.

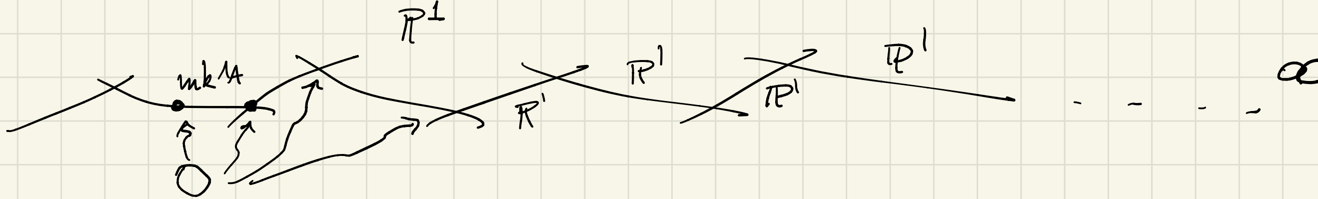
Flip:



¡PARA! $\swarrow$ $\frac{1}{m}(1,q)$

Flip $\downarrow$ único P-resol. extremal

Familia universal de Antiflips

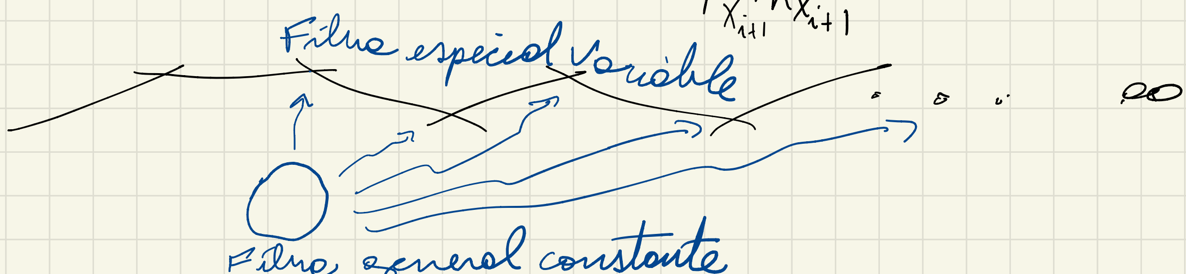


Resulta que $\quad n_{i-1} + n_{i+1} = \delta\, n_i$

$$\frac{n_{i+1}}{n_i} = \delta - \frac{1}{\frac{n_i}{n_{i-1}}} \quad \therefore \quad \delta - \frac{1}{\delta - \frac{1}{\delta - \cdots}} = \frac{\delta + \sqrt{\delta^2 - 4}}{2}$$

Lo que es irracional si $\delta \geq 3$: $\quad \frac{n_{i+1}}{n_i} \xrightarrow{i \to \infty} \frac{\delta + \sqrt{\delta^2 - 4}}{2}$

$$\frac{\Gamma_{x_i}^{-} \cdot K_{x_i}}{\Gamma_{x_{i+1}}^{-} \cdot K_{x_{i+1}}} = \frac{n_{i+1}}{n_i}$$



Ej1.- $\frac{1}{11}(1,3)$ $\qquad \frac{11}{3} = 4 - \frac{1}{3}$ $\qquad \frac{1}{4}(1,1)$ P-resol. extremal

$3 - [4]$

Familia de Antiflips

$$\underbrace{\phi - \underbrace{[\overset{3}{\bar{2}}, \overset{3}{5}, \overset{1}{3}]}_{mk1A} - \underbrace{[2,3,\bar{2},2,7,4]}_{mk1A}}_{mk2A} - \underbrace{[2,3,2,2,2,\bar{2},5,7,3]}_{mk1A} - \infty$$