UC WORKSHOP
TITLES & ABSTRACTS

Michela Artebani (U. Concepción)
Title: *Calabi-Yau hypersurfaces in toric $\mathbb{Q}$-Fano varieties*
Abstract: A large class of examples of Calabi-Yau varieties can be obtained considering anticanonical hypersurfaces in toric Fano varieties. A special interest for such families of Calabi-Yau's arised after the work of Batyrev, who defined a duality between them which satisfies the requirement of topological mirror symmetry:

$$h^{p,q}(X) = h^{n-p,q}(X^*), \quad 0 \leq p, q \leq n = \dim(X),$$

where X, X^* are general elements in the dual anticanonical linear series. A different mirror construction has been given by Berglund, Hübsch and for certain Calabi-Yau hypersurfaces in fake weighted projective spaces.
In this talk we will present a duality between families of Calabi-Yau hypersurfaces in $\mathbb{Q}$-Fano toric varieties which generalizes the previous ones. This is based on a duality between pairs (Δ_1, Δ_2) of polytopes, where Δ_1 is the Newton polytope of the family and Δ_2 is the anticanonical polytope of the ambient toric variety. Batyrev construction corresponds to the case when $\Delta_1 = \Delta_2$ is reflexive, while the BHK construction appears when Δ_1, Δ_2 are simplices.
This is joint work with Paola Comparin and Robin Guilbot

Robert Auffarth (U. Chile)
Title: *The Kodaira dimension of $\mathcal{A}_6$*
Abstract: Let $\mathcal{A}_g$ be the moduli space of principally polarized abelian varieties. It is well-known that $\mathcal{A}_1$ and $\mathcal{A}_2$ are rational, Clemens showed that $\mathcal{A}_4$ is unirational, and work by Donagi, Mori, Mukai and Verra shows that $\mathcal{A}_5$ is unirational. For larger values of g, Freitag (for $24 \mid g$), Tai ($g \geq 9$) and Mumford $g \geq 7$ constructed effective divisors of small slope on $\mathcal{A}_g$, and therefore proved that for these values of g, $\mathcal{A}_g$ is of general type. The problem that remains is to find the Kodaira dimension of $\mathcal{A}_6$. In this talk we will introduce the problem as well as possible ways of attacking it.

Indranil Biswas (TIFR)
Title: *Isomonodromic deformations of logarithmic connections and stability*
Abstract: Let X_0 be a compact connected Riemann surface of genus g with $D_0 \subset X_0$ an ordered subset of cardinality n, and let E_G be a holomorphic principal G-bundle on X_0, where G is a complex reductive affine algebraic group, that admits a logarithmic connection ∇_0 with polar divisor D_0. Let $(\mathcal{E}_G, \nabla)$ be the universal isomonodromic deformation of (E_G, ∇_0) over the universal Teichmüller curve $(\mathcal{X}, \mathcal{D}) \to Teich_{g,n}$, where $Teich_{g,n}$ is the Teichmüller space for genus g Riemann surfaces with n-marked points. We prove the following: Assume that $g > 1$ and $n = 0$. Then there is a closed complex analytic subset $\mathcal{Y} \subset Teich_{(g,n)}$, of codimension at least g, such that for any $t \in Teich_{(g,n)} \setminus \mathcal{Y}$, the principal G-bundle $\mathcal{E}_G|_{\mathcal{X}_t}$ is semistable, where $\mathcal{X}_t$ is the compact Riemann surface over t. Assume that $g > 0$, and if $g = 1$, then $n > 0$. Also, assume that the monodromy representation for ∇_0 does not factor through some proper parabolic subgroup of G. Then there is a closed complex analytic

subset $\mathcal{Y}' \subset Teich_{(g,n)}$, of codimension at least g, such that for any $t \in Teich_{(g,n)} \setminus \mathcal{Y}'$, the principal G-bundle $\mathcal{E}_G|_{\mathcal{X}_t}$ is semistable. Assume that $g > 1$. Assume that the monodromy representation for ∇_0 does not factor through some proper parabolic subgroup of G. Then there is a closed complex analytic subset $\mathcal{Y}" \subset Teich_{(g,n)}$, of codimension at least $g-1$, such that for any $t \in Teich_{(g,n)} \setminus \mathcal{Y}'$, the principal G-bundle $\mathcal{E}_G|_{\mathcal{X}_t}$ is stable.

Elizabeth Gasparim (U.C. del Norte)
Title: *Adjoint orbits and Lefschetz fibrations*
Abstract: I will describe geometric aspects of the structure of symplectic Lefschetz fibrations on adjoint orbits of semisimple Lie groups, together with examples and several open questions.

María del Rosario González-Dorrego (UAM)
Title: *On singular varieties with smooth subvarieties*
Abstract: Let k be an algebraically closed field of characteristic 0. Let Z be a reduced irreducible nonsingular subvariety of a normal n-fold with certain type of singularities, such that $Z \cap Sing(X) \neq \emptyset$. We study the singularities of X through which Z passes.

Stephen Griffeth (U. Talca)
Title: *Double affine Hecke algebras and spaces of conformal blocks*
Abstract: We explain how to realize the double affine braid group of a reductive complex algebraic group G as the fundamental group of a certain open subset of the moduli stack parametrising principal G-bundles on elliptic curves, and hence to give a conceptual explanation for a beautiful observation of Cherednik: for certain specialisations of the double affine Hecke algebra (DAHA), the Verlinde algebra, defined via fusion product of representations of a central extension of the loop group of G, is a quotient of the spherical induced representation of the DAHA. (This is joint work in progress with Jethro van Ekeren.)

Antonio Laface (U. Concepción)
Title: *On del Pezzo elliptic varieties*
Abstract: A Mori dream space is a normal projective variety with finitely generated Cox ring. It is a well known result that a log Fano variety is a Mori dream space. Log Fano varieties include toric varieties, whose Cox ring is a polynomial ring, and Fano varieties, which have ample anticanonical class. It is natural to ask what happens if one relaxes the condition about the anticanonical class by requiring it to be semiample of Iitaka codimension one. Examples of varieties with this property are certain blowing-ups of del Pezzo varieties, such as cubic hypersurfaces or complete intersections of two quadrics. More precisely, given an n-dimensional del Pezzo variety Y and a linear space L which intersects Y along a zero-dimensional subscheme, the resolution of indeterminacy of the linear projection $Y \to \mathbb{P}^{n-1}$ from L gives an elliptic fibration

$$\pi\colon X \to \mathbb{P}^{n-1},$$

where X is the blowing-up of Y at $L \cap Y$. Our work consisted in determining all the possible Mordell-Weil groups for π and showing that such a group is finite if and only if X is a Mori dream space.

This is joint work with A. L. Tironi, L. Ugaglia.

Max Leyton (U. Talca)
Title: *Deformando los Espacios de m-jets de singularidades aisladas de hipersuperficies*

Abstract: Sea $\mathbb{K}$ un cuerpo algebraicamente cerrado, $W \to S$ un morfismo de $\mathbb{K}$-esquemas y consideremos el *Funtor de m-jets Relativo*: $F_m : S\text{-}\mathcal{E}sq \to \mathcal{C}onj$; $Y \mapsto \mathrm{Hom}_S(Y \times_{\mathbb{K}} \mathrm{Spec}\,\mathbb{K}[t]/(t^{m+1}), W)$, donde $S\text{-}\mathcal{E}sq$ y $\mathcal{C}onj$ son la categoría de S-esquemas y la categoría de conjuntos respectivamente. Es sabido que el funtor F_m es representable por un S-esquema, el cual denotaremos por $\mathrm{W(S)}_m$ y llamaremos *Espacio de m-jets Relativo.* Si V es una variedad sobre $\mathbb{K}$, entonces el esquema $\mathrm{V(Spec\,\mathbb{K})}_m$ es el clásico *Espacio de m-jets*, el cual notaremos por V_m.

Los Espacios de m-jets Relativos satisfacen la siguiente propiedad de cambio de base: sea $S' \to S$ un morfismo y $W' := W \times_S S'$, entonces $\mathrm{W'(S')}_m \cong \mathrm{W(S)}_m \times_S S'$ sobre S'. Esta propiedad nos dice que para todo punto s en S se tiene que $(W_s)_m \cong \mathrm{W(S)}_m \times_S \{s\}$, donde W_s es la fibra sobre $s \in S$.

Una de las principales motivaciones de estudiar los espacios V_m proviene del hecho que la geometría de estos espacios mantiene una estrecha relación con la geometría local del lugar singular, $\mathrm{Sing}\,V$, de la variedad V. Bajo esta motivación, es natural plantearnos la siguiente pregunta: ¿Si $W \to S$ es una deformación de una variedad V, entonces que el morfismo $\mathrm{W(S)}_m \to S$ es una deformación del esquema V_m? No es difícil construir ejemplos donde esta pregunta tiene una respuesta negativa. Sin embargo, existen muchas familias de variedades donde la pregunta tiene una respuesta afirmativa, por ejemplo si V es localmente intersección completa con singularidades a lo más de tipo log-canónica. Como los Espacios de m-jets dependen fuertemente de las propiedades locales de $\mathrm{Sing}\,V$, entonces de forma intuitiva podríamos pensar que una deformación que "cambie demasiado" la topología en torno de $\mathrm{Sing}\,V$ tiene una fuerte posibilidad de no inducir una deformación de los espacios de m-jets. Una μ-deformación (donde μ es el número de Milnor) de una singularidad aislada de hipersuperficie V es un ejemplo de deformación que "no cambia demasiado" las propiedades locales en torno de $\mathrm{Sing}\,V$. Entonces es razonable plantearse la pregunta anterior en el caso que $W \to S$ es una μ-deformación de una hipersuperficie V con un única singularidad aislada. Cabe destacar que a pesar que las μ-deformaciones de V han sido fuertemente estudiadas, aún quedan varios problemas abiertos

Supongamos que la hipersuperficie V es definida por un polinomio f irreducible. Recientemente probamos que para ciertas deformaciones de V, por ejemplo las $\Gamma(\mathrm{f})$-deformaciones, inducen una deformación de la estructura reducida del Espacio de m-jets. Cabe señalar que una $\Gamma(\mathrm{f})$-deformación es una μ-deformación de V, más aún si f es un polinomio casi-homogéneo el reciproco es valido.

En esta charla daremos más detalles de este problema y explicaremos la ideas fundamentales de la demostración del resultado anteriormente citado.

<u>**Ricardo Menares**</u> (U.C. Valparaiso)

Title: *On the essential minimum of Faltings height*

Abstract: Height functions are designed to measure the size of arithmetic objects. The simplest example is Weil's height: for a given rational number $x = a/b$, the Weil height attaches the value

$$\log \max\{|a|, |b|\},$$

which, loosely speaking, corresponds to the number of digits necessary to write down x. More generally, x can be taken to be an algebraic number and the Weil height of x is a nonnegative real number that indicates how big are the coefficients of the corresponding minimal polynomial.

In this talk we will focus in the case of Faltings height, that measures the size of elliptic curves defined over number fields. Loosely speaking, it takes into account the size of the locus of bad reduction (arithmetic information) and of the periods of the curve (transcendental information).

The elliptic curve of smallest Faltings height is the one with j-invariant equal to zero. We will show that this value of the height is isolated and that the first accumulation point (the so-called essential minimum) lies in between the heights of the elliptic curves with j-invariant 1 and -1.

This is joint work with José Burgos Gil and Juan Rivera-Letelier.

Anita Rojas (U. Chile)

Title: *Families of completely decomposable Jacobian varieties and special subvarieties of* $\mathcal{A}_g$

Abstract: Let G be a finite group acting on genus g with signature $m = [0; s_1, \ldots, s_r]$, and generating vector $\nu = (g_1, \ldots, g_r)$. For a fixed pair (m, ν), and by moving the branch points of the covering in $\mathbb{P}^1$, one obtains an $(r-3)-$dimensional family of such coverings, and a corresponding $(r-3)-$dimensional family of Jacobians $\mathcal{J}(G, m, \nu)$.

On the other hand, the symplectic group $\mathrm{Sp}(2g, \mathbb{Z})$ acts on the Siegel upper half space $\mathbb{H}_g$, and $\mathcal{A}_g = Sp(2g, \mathbb{Z}) \setminus \mathbb{H}_g$ is a complex analytic space which parametrizes principally polarized abelian varieties of dimension g up to isomorphism.

Denote by $Z(G, m, \nu)$ the closure of $\mathcal{J}(G, m, \nu)$ in $\mathcal{A}_g$. The action of G on the curve X, hence on its Jacobian JX, induces a symplectic representation $\rho : G \to Sp(2g, \mathbb{Z})$ of G. Let $\mathbb{H}_g^G$ be the set of fixed points of G in $\mathbb{H}_g$. Frediani-Ghigi-Penegini give a nice characterization of when $Z(G, m, \nu)$ is a *special subvariety*. Their criterion is as follows, if the dimension of $\mathbb{H}_g^G$ equals the dimension of $\mathcal{J}(G, m, \nu)$, which is $r-3$, then $Z(G, m, \nu)$ is a special subvariety of $\mathcal{A}_g$ that it is contained in the closure $\mathcal{T}_g$ of the Torelli (or Jacobian) locus, and which intersects non-trivially the Torelli locus $\mathcal{T}_g^0$.

Additionally, in their *Handbook of Moduli* article, Moonen and Oort ask about the existence of positive dimensional special subvarieties Z of $\mathcal{T}_g$ such that the abelian variety corresponding with the geometric generic point of Z is isogenous to a product of elliptic curves.

Given a pair (m, ν) for a fixed G, using results of Behn-Rodriguez-Rojas, one can find the dimension of $\mathbb{H}_g^G$, although it is computationally expensive. Additionally, the above cited work of Frediani-Ghigi-Penegini provides a code which can compute the dimension for low genus examples as well.

In joint work work with Paulhus, motivated by a different (although related) question posed by Ekedahl and Serre, we develop several examples of completely decomposable Jacobian varieties in different dimensions, including several families. We provide in this way a good scenario where to find families of Jacobian varieties that might give rise to special subvarieties.

In the first half of this talk we will present the background and some of the results regarding families of completely decomposable Jacobian varieties. In the second half, we will address what it is a work in progress. We will explain the ideas involved in finding special subvarieties, we will show some examples of special subvarieties corresponding to families of completely decomposable Jacobian varieties, and some ideas/questions to continue the research in this direction.

Mark Spivakovsky (U. Paul Sebatier)

Title: : *On the work of John Nash in geometry*

Abstract: : In this talk we will attempt to summarize John Nash's main contributions to geometry, spanning the nearly two decades from 1950 to 1968. We will concentrate on the following results and constructions:

(1) Nash's theorem on embedding differential manifolds as connected components of real algebraic manifolds.
(2) Two versions of the celebrated Nash embedding theorem. The first, the Nash–Kuiper theorem, asserts that any non-expanding embedding of an m-dimensional C^1-manifold into an n-dimensional Euclidean space with $n > m$ can be approximated arbitrarily well by an isometric C^1 embedding. In Nash's original result n was greater than or equal to $m + 2$, but Kuiper improved the bound to $n > m$. The second version, much more difficult to prove and also published in the Annals of Mathematics, says that for k between 3 and infinity any m-dimensional C^k-manifold M can be isometrically C^k-embedded into an n-dimensional Euclidean space, where $n?m(3m + 11)/2$ if M is a compact manifold and $n?m(m + 1)(3m + 11)/2$ if M is a non-compact manifold.

Most of the lecture will be devoted to Nash's later contributions from the nineteen sixties:
(3) Nash blowing up as a conjectural method for constructing a canonical resolution of singularities of varieties in characteristic zero.
(4) The Nash problem on the spaces of arcs on singular algebraic varieties.

Giancarlo Urzúa (U.C. Chile)

Title: *A missing ingredient to give a proof of Kawamata-Viehweg vanishing theorem for rational surfaces*

Abstract: We can prove density of Chern slopes in [2,infinity[for étale-simply connected surfaces of general type with Picard scheme equal to a reduced point (arXiv:1402.5801). It turns out that its proof gives a possible strategy to prove the Kawamata-Viehweg vanishing theorem for rational surfaces. I will discuss details around a specific missing ingredient, and how it effectively works in the proof of the above mentioned reduced density.