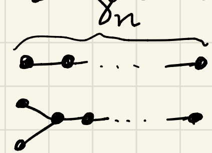
SGA 5

14/9/20

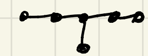
given

<u>Hoy</u> : Rouleau-Rousseau Duke.

X = superficie proy. $/\mathbb{C}$ con singularidades ADE-RDP-Du Val-Klein sing



E_6, E_7, E_8



$$\begin{array}{l} Y \xrightarrow[\text{min}]{\text{resol.}} X \xleftarrow{\pi} \mathcal{X} = \text{orbifold.} \\ \cup \\ E = \text{div excepcional} \end{array}$$

$a_n = \#\left\{\begin{array}{l}\text{sing } A_n \\ \text{en } X\end{array}\right\}$ ...

Tenemos $c_1^2(Y) = K_Y^2 = K_{\mathcal{X}}^2 = c_1^2(\mathcal{X})$ y

$$c_2(\mathcal{X}) = c_2(Y) - \sum (n+1)(a_n + d_n + e_n) + \sum \frac{a_n}{n+1} + \frac{d_n}{4(n-2)} + \frac{e_6}{24} + \frac{e_7}{48} + \frac{e_8}{120}$$

$$= c_2(X) - \sum \left(1 - \frac{1}{n+1}\right) a_n - \dots$$

$$c_1^2(Y) = c_1^2(X)$$

$$c_2(Y) = c_2(X) + \sum n(a_n + d_n$$

$E_6 + E_7$

$(E-)$

Clase de Segre $= s_2 = c_1^2 - c_2$.

Thm 1: Si $s_2(Y)+s_2(\mathcal{X})>0 \Rightarrow \Omega^1_Y$ es big.
(y así Y es cuasi-alg. hiperbólico)

Bogomolov

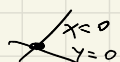
De Oliveira

$$\left[\, s_2(Y)+s_2(\mathcal{X})>0 \iff 2s_2(Y)+\sum\left(n+1-\frac{1}{n+1}\right)a_n+\sum\left(n+1-\frac{1}{4(n-2)}\right)d_n+\cdots>0 \,\right]$$

Obs|- Bogomolov: $s_2(Y)>0 \Rightarrow \Omega^1_Y$ es big.
$\frac{c_1^2}{c_2}\in\,]1,3] \Rightarrow$ cuasi alg hip.

Thm 2: Si $X\subseteq \mathbb{P}^3$ hipersuperficie de grado d y ℓ sing. A_k, entonces $\ell > \frac{4(k+1)}{k(k+2)}(2d^2-5d) \Rightarrow \Omega^1_Y$ big.

Thm 3: $X \xrightarrow{d:1} \mathbb{P}^2$
$\qquad\qquad\quad \cup$
$\qquad\qquad\quad D=\sum_{j=1}^{k} D_j$ SNC de grados d_j, $\quad d=\sum_{j=1}^{k} d_j$

$(z^d=xy)$ 

$\cong A_{d-1}$

(así la X sólo tiene singularidades A_{d-1}) Asumir $d_j \geq c\ \forall j$

$\Rightarrow \Omega^1_Y$ big si $k(k-1) > \frac{8d^2(2d-5)}{c^2(d^2-1)}$.

Teo: Si $s_2(Y) + s_2(\mathcal{X}) > 0 \Rightarrow h^0(Y, S^m \Omega^1_Y) \geq \left(\frac{s_2(Y) + s_2(\mathcal{X})}{12}\right) m^3 + O(m^2)$

y así Ω^1_Y es big.

Dem: Consideramos:

$$0 \to \Omega^1_Y \to \Omega^1_Y(\log E) \to \bigoplus_{E_i \in E} \mathcal{O}_{E_i} \to 0$$

$$(*) \quad 0 \to S^m \Omega^1_Y \to S^m \Omega^1_Y(\log E) \to Q_m \to 0$$

donde Q_m está soportado sobre E.

obs: Sabemos por Miyaoka $H^0(Y \setminus E, S^m \Omega_Y) = H^0(Y, S^m \Omega_Y(\log E))$.

"orbifold view": Bogomolov Vanishing $H^0(\mathcal{X}, S^m \Omega^{1\vee}_{\mathcal{X}} \otimes K_{\mathcal{X}}^{\otimes p}) = 0$ si $m > 2p$.

\+ R-R: $h^0(\mathcal{X}, S^m \Omega^1_{\mathcal{X}}) \geq \frac{s_2(\mathcal{X})}{6} m^3$.

Clave: $h^0(Y \setminus E, S^m \Omega^1_Y) = h^0(\mathcal{X}, S^m \Omega^1_{\mathcal{X}})$

Por $(*)$: $$\frac{h^0(S^m\Omega^1_Y)}{m^3} \geq \frac{h^0(S^m\Omega^1_Y(\log E))}{m^3} - \frac{h^0(Q_m)}{m^3} \quad (\mathrm{I})$$

El problema es el aporte de $h^0(Q_m)$

Miyaoka: $$0 \to S^m\Omega^1_Y \otimes K_Y^{1-m} \to S^m\Omega^1_Y(\log E)\otimes K_Y^{1-m} \to Q_m \to 0 \quad (**)$$

Mas aun:

orb2: $$h^0(S^m\Omega^1_Y(\log E)\otimes K^{(1-m)}) \underset{\text{orbi}}{\simeq} h^0(Y\setminus E, S^m\Omega^1_Y\otimes K_Y^{(1-m)}) \simeq h^0(X, S^m\Omega^1_X \otimes K_X^{1-m}) \underset{\text{Serre}}{\simeq} h^2(X, S^m\Omega^1_X) \underset{\text{Bogom.}}{=} 0 \quad \text{si } m>2$$

$$\left(\Omega^{n-p} \simeq (\Omega^p)^\vee \otimes K \underset{\substack{n=2\\p=1}}{\Longrightarrow} \Omega^{1\vee} \simeq \Omega^1\otimes K^{-1}\right)$$

¿serán orb1 y orb2 posibles sin orbifolds?

$$\therefore \; H^0(Q_m) \subset H^1(S^m\Omega^1_Y \otimes K^{(1-m)}) \underset{\text{Serre}}{=} H^1(S^m\Omega^1_Y) \text{ y}$$

tenemos por Bogomolov $h^2(S^m\Omega^1_Y) = 0$.

$$\Rightarrow \quad h^0(S^m\Omega^1_Y) = \chi(S^m\Omega^1_Y) + h^1(S^m\Omega^1_Y)$$

(II)

$$\Rightarrow \quad \frac{h^0(S^m\Omega^1_Y)}{m^3} \geq \frac{s_2(Y)}{6} + \frac{h^1(S^m\Omega^1_Y)}{m^3} \geq \frac{s_2(Y)}{6} + \frac{h^0(Q_m)}{m^3}$$

(I) y (II)

$$\frac{h^0(S^m\Omega^1_Y)}{m^3} \geq \frac{s_2(X)}{6} - \frac{h^0(Q_m)}{m^3} + O\left(\frac{1}{m}\right)$$

$$+$$

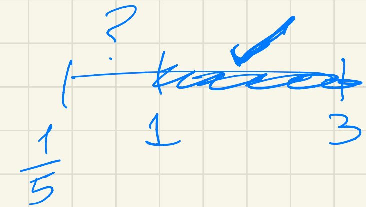
$$\frac{h^0(S^m\Omega^1_Y)}{m^3} \geq \frac{s_2(Y)}{6} + \frac{h^0(Q_m)}{m^3}$$

$$h^0(S^m\Omega^1_Y) \geq \frac{\left(s_2(X) + s_2(Y)\right)}{12} m^3 + O(m^2)$$

∎

Geografía de sup. con Ω^1_X big.

- Ω^1 amplio $\Rightarrow c_1^2 > c_2$ (Fulton–Lazarsfeld).
- $c_1^2 > c_2 \Rightarrow \Omega^1$ big
- $S \subset \mathbb{P}^3$ hiper sup suave $\Rightarrow h^0(S^m\Omega^1) = 0$.



¿Será denso?

Prop: $s_2(Y) + s_2(\mathcal{X}) > 0 \Rightarrow \frac{c_1^2}{c_2} \in \left]\frac{3}{5}, 3\right]$.

Dem: Manipular numeros y BMY $\mathcal{X} \Leftrightarrow \Omega^1_Y(\log E)$

obs: Si $s_2(Y) + s_2(\mathcal{X}) > 0$

$$\Rightarrow \frac{c_2(Y)}{c_1^2(Y)} + \frac{c_2(\mathcal{X})}{c_1^2(\mathcal{X})} < 2$$

Lonergan
«Logar. orb. Euler numbers of surf. with Applic.»
PLMS 2003

$$\frac{c_1^2(Y)}{c_2(Y)} \to \frac{3}{5} \qquad \frac{c_1^2(\mathcal{X})}{c_2(\mathcal{X})} \to 3$$

hay pocos ejemplos con $\frac{c_1^2}{c_2}(\mathcal{X}) \to 3$

FIN.